Training and Research Activities Report
First Year

PhD in Information Technology and Electrical Engineering
Cycle XXIX

Christian Erazo Ordoez
Tutor: Mario di Bernardo

Università degli Studi di Napoli
Federico II

Contents

I General Information

Christian Camilo Erazo Ordoez
Master in Engineering - Industrial Automation
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I was admitted for the program: **PhD in Information Technology and Electrical Engineering, Cycle** 29. I am receiving support from a fellowship provided by *Università degli Studi di Napoli Federico II*. My research activities are supervised by Professor **Mario di Bernardo**.

II Study and Training activities

II-A Courses

- Name: Prof. John Hogan, Theory and applications of piecewise smooth systems
  Location: Univ. of Naples Federico II, Naples, Italy.
  Date: 16-20, June 2014.
  Number of Hours: 15.
  Credits: 5.

- Name: Prof. Stephen Boyd, Convex Optimization.
  Location: Universit degli studi del Sannio, Benevento.
  Date: 23, 24 June 2014.
  Number of Hours: 7
  Credits: 2

- Name: Prof. Henning Schulzrinne, Three core issues for the Internet: things, security and economics
  Location: Univ. of Naples Federico II, Naples, Italy.
  Date: 19, 20 February 2014.
  Number of Hours: 8
  Credits: 2.

II-B Seminars

- Name: Seminars of research group SINCRO.
  Location: Univ. of Naples Federico II, Naples, Italy.
  Date: August 2014 to December 2014.
  Number of Hours: 8.
  Credits: 1.

III Algorithm for computing domains of attraction in planar switched linear systems

Computing domains of attraction of equilibria plays an important role in control systems design, since it is not sufficient to determine if a given system has a stable behavior but also it is essential to guarantee the system trajectories remain within some desired operation region. Most of the results on estimation of domains of attraction are based on the use of Lyapunov functions. Examples include electrical systems and mechanical systems [1, 2, 3], recent studies in biological systems, in particular for analyzing the tumor growth dynamics and resilience problem in ecosystems [4, 5]. The problem of estimating domains of attraction (DA) in switched linear systems is still open. Most difficulties come from the presence of discontinuities and nonlinearities [6]. In the literature, there exist relatively few studies mostly dealing with discrete-time saturated systems [7, 8, 9] using polyhedral Lyapunov functions. In [10] it is proposed to estimate DA for continuous-time saturated systems by computing polyhedral Lyapunov functions for the Euler approximating system. In [11], the authors decompose the continuous-time saturated linear system into different switched systems, each subsystem being associated to a different polyhedral set whose intersection with the others provides an estimate of domain of attraction.

The aim of my work is to derive a simple approach that can be used to compute numerically DA in planar switched linear systems. The methodology starts from an initial estimate provided by a polyhedral function $V(x)$, then the set is decomposed and enlarged by verifying the direction of the vector field along the boundary of the set. Specifically, we are interested in studying whether the velocity vector points inwards on all boundaries of the closed set, in order to establish whether the region of interest is invariant with respect to the system trajectories.

III-A Research description

The approach considers an initial polyhedral set defined as $\wp = \{x : Wx \leq 1\}$ which contains the origin. Then every line segment s_i that encloses the polyhedral set is displaced along its normal direction, verifying at every iteration that $s_i \cdot f(x) < 0$, guaranteeing that the velocity vector is pointing inwards. Moreover, extra line segments are added in order to close the set after displacement. Those segments are parametrized from the vertices of the initial polyhedral set. This approach can be implemented following the algorithm given below

Algorithm 1 Computation of domain of attraction of constrained switched systems

Input : $f(x)$ and $\wp$.
Output :$\wp^{max}$ (Domain of attraction).
1. Let
$$\wp = \{x : Wx \leq 1\}$$
2. Shift every line segment of the set along its normal direction and compute the new segments
$$s_N = (1-t)P_0 + tP_1$$
3. Verify
$$s_i \cdot f(x) < 0 \tag{1}$$
if (1) is met, go to step 2., else set $\wp = \wp^{max}$

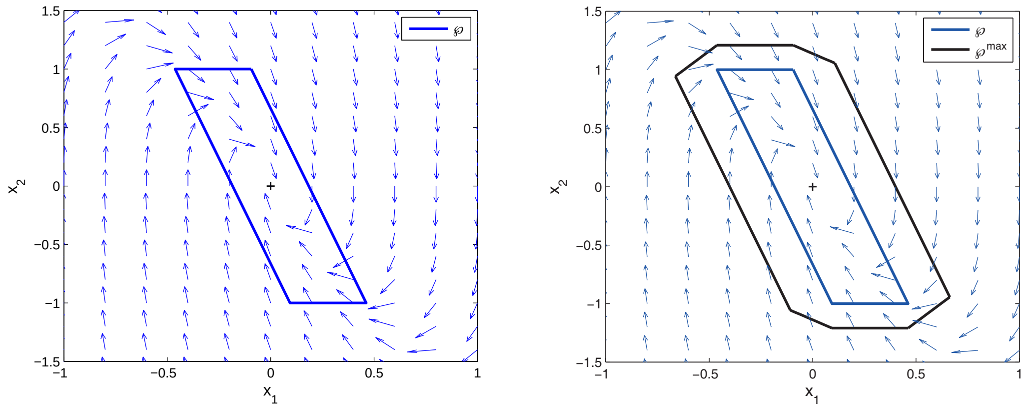
To illustrate the methodology, we consider a single input continuous-time saturated system described by:

$$\begin{aligned} \dot{x} &= Ax + Bsat(u) \\ u &= Fx \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input and sat(.) : $\mathbb{R} \to \mathbb{R}$ is the saturation function:

$$\text{sat}(u) = \begin{cases} 1 & Fx > 1 \\ Fx & -1 \leq Fx \leq 1 \\ -1 & Fx < -1 \end{cases} \tag{3}$$

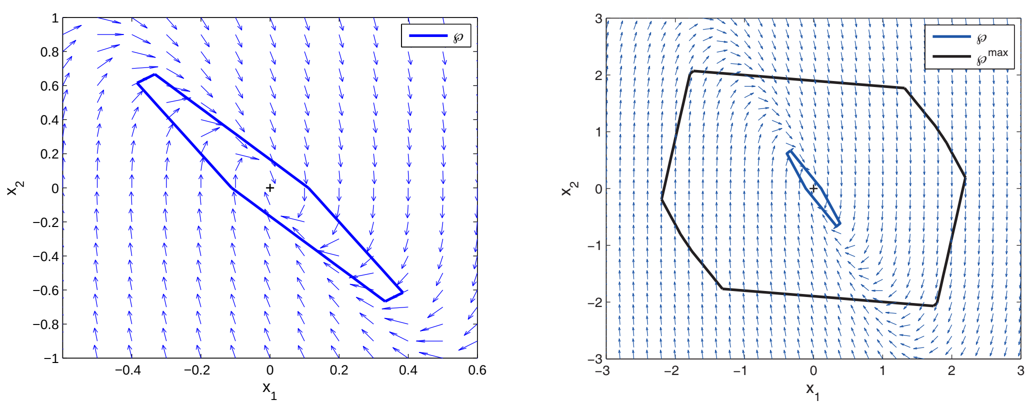
Figure 1(b) shows the estimation of DA, for an initial 0-symmetric polyhedral set computed by similarity transformation [12]. The approach seems to be effective, nevertheless the estimation is still conservative.



(a) Initial estimate   (b) Enlarging Domain of attraction

Figure 1: Illustration of the methodology proposed

Now, we consider an initial polyhedral set derived by recursion method [8]. As it can be seen in Figure 2(b) increasing the number of sides of the initial set, the estimation is less conservative.



(a) Initial estimate   (b) Enlarging Domain of attraction

Figure 2: Illustration of the methodology proposed

III-B Conclusions and Future Work

We have investigated the domain of attraction in saturated switched systems by checking the direction of the vector field in each subsystem. The results show the effectiveness of our strategy dealing with the saturated switched systems. In particular we observed that it is possible to estimate numerically the invariant region around an equilibrium by analyzing how the vector field direction changes along the boundaries of an initial set that is then enlarged iteratively using this information. So far, the methodology has been evaluated in saturated switched systems, we want to extend this study to switched systems with discontinuities in the control law. In addition, we want to include a better an more refined estimate of the region of attraction by considering different methodologies to iteratively expand the region of interest.

Bibliography

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[3] F. Amato, F. Calabrese, C. Cosentino, and A. Merola, "Stability analysis of nonlinear quadratic systems via polyhedral lyapunov functions," *Automatica*, vol. 47, no. 3, pp. 614–617, 2011.

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[10] M. Demenkov, "Estimating basin of attraction in piecewise-linear systems by nonsmooth lyapunov functions," in *Internation Meeting on Analysis and Applications of Nonsmooth Systems*, 2014.

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[12] H. Kiendl, J. Adamy, and P.Stelzner, "Vector norms as lyapunov functions for linear systems," *IEEE trans. on Aut. Contr.*, vol. 37, pp. 839–842, 1992.

IV Products

IV-A Publications

- C. Erazo and M. di Bernardo (2015), An algorithm for computing domains of attraction in planar switched systems. (in preparation)

V Conferences and Seminaries

V-A Presentation

- Title: Adaptive Technique for Estimating Basins of Attractions
  Event: Seminar of research group SINCRO.
  Location: Univ. of Naples Federico II, Naples, Italy.
  Date: December 2014.

VI Attachments

VI-A Credits Summary Report

Student: Christian Erazo
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Cycle XXIX

	Credits year 1								**Credits year 2**								**Credits year 3**									
	Estimated	1 bimonth	2 bimonth	3 bimonth	4 bimonth	5 bimonth	6 bimonth	**Summary**	**Estimated**	1 bimonth	2 bimonth	3 bimonth	4 bimonth	5 bimonth	6 bimonth	**Summary**	**Estimated**	1 bimonth	2 bimonth	3 bimonth	4 bimonth	5 bimonth	6 bimonth	**Summary**	**Total**	**Check**
Modules	**18**		7				2	**9**	**9**							**0**								**0**	**9**	**30-70**
Seminars	**13**					1		**1**	**6**	3	3		3		6	**15**								**0**	**16**	**10-30**
Research	**34**	0	0	0	0	0	0	**0**	**42**	7	7	7	7	7	7	**42**								**0**	**42**	**80-140**
	65	0	7	0	0	1	2	**10**	**57**	10	10	7	10	7	13	**57**	**0**	0	0	0	0	0	0	**0**	**67**	**180**

Year	Lecture/Activity	Type	Credits	Certification	Notes
1	Theory and applications of piecewise smooth systems	Course	5	x	
1	Convex Optimization	Course	2	x	
1	Three core issues for the Internet: things, security and economics	Course	2	x	
1	Seminars of research group SINCRO	Seminar	1	x	