Mariano Pascale

Tutor: Carlo Forestiere

XXXIII Cycle - III year presentation

Material-Independent Modes for the Electromagnetic Scattering: from Quasistatic to Full-Wave Formulations

Università degli Studi di Napoli Federico II

General Information

- MSc in Electronic Engineering, Università degli studi di Napoli Federico II (2016)
- Cooperation Associate, Beams Department, CERN (2017)
- Athenaeum fellowship (2018-present). Tutor: Prof. Carlo Forestiere

YEAR	COURSES	SEMINARS	RESEARCH
I	10.3	5.2	44.5
II	23.4	6.9	29.7
III	6	5.2	59.8

- Study and Training Activities: **12** courses, **1** language course, **3** schools, **29** seminars
- Scientific production: **7** journal papers (+2 under preparation), **2** conference papers, **2** book chapters
- Period abroad: 6 months at the Advanced Science Research Center, City University of New York (CUNY), supervised by Prof. Andrea Alù. (+1 year collaboration in smart working)

Course List (13)

- Approssimazione di problemi alle derivate parziali e applicazioni (1st year)
- Plasmonics and Metamaterials (1st year)
- Ciberconflitti sicurezza informatica, difesa, stabilità internazionale e diritto umanitario (1st year)
- Mathematical and Numerical Models for Multi-physics Applications (1st year)
- Elettromagnetismo e relatività (2nd year)
- A leap into Functional Data Analysis: from theory to applications (2nd year)
- MHD Equilibrium and Stability (2nd year)
- Data science and optimization, M. Gaudioso, L. Palagi, E. (2nd year)
- Strategic Orientation for STEM Research & Writing (2nd year)
- Artificial Intelligence for Energy and Environmental Systems (2nd year)
- English course (C1) (2nd year)
- Introduction to Parallel Computing with MPI and OpenMP (2nd year)
- Introduction to nanotechnology (3rd year)

PhD School List (3)

- International School of Plasmonics and Nano-Optics , Cetraro (1st year)
- Ferdinando Gasparini XXII edizione, Napoli (2nd year)
- European School on Metamaterials (on-line), 14th International Congress on Artificial Materials for Novel Wave Phenomena, Metamaterials 2020 (3rd year)

Seminar List (29) 1/2

- IBM Q: building the first universal quantum computers for business and science - Federico Mattei and Najla Said (1st year).
- The Power of Trefftz Approximations: Applications in Electromagnetics - Igor Tsukerman (1st year).

- Non-Asymptotic and Nonlocal Homogenization of Periodic Electromagnetic Structures - Igor Tsukerman (1st year).
- Tailoring waves at the extreme with metamaterials - Nader Engheta (1st year).
- Near-zero-index photonics - Nader Engheta (1st year).
- Tomografia e Imaging: Principi, Algoritmi e Metodi Numerici - Pasquale Memmolo (1st year).
- Computational and machine learning methods for complex ecosystems - Edoardo Pasolli (2nd year).
- Chaos in magnetization dynamics - Claudio Serpico (2nd year).
- Spin-orbit optical phenomena - Lorenzo Marrucci (2nd year).
- IEEEXplore Training and Autorship Workshop - Eszter Lucacks (2nd year).
- Robotics in medical applications: an overview of the current medical robotics market from the industry's point of view - Vincenzo Schettino (2nd year).
- Medical thermal therapy and monitoring using microwave inverse scattering - Mahta Moghaddam (2nd year).
- Designer matter: meta-material interactions with light, radiowaves and sound - Andrea Alù (2nd year).
- The ASRC @ 5: Showcasing Interdisciplinary Excellence, Advanced Science Research Center (ASRC), 85 St. Nicholas Terrace, New York (2nd year).
- Synthetic interfacial optics with metasurfaces and 2D monolayers, Cheng-Wei Qiu - National University of Singapore, Advanced Science Research Center (ASRC), New York. (2nd year).

Seminar List (29) 2/2

- On the Hall effect in three-dimensional metamaterials", Christian Kern (University of Utah), at the Advanced Science Research Center, City University of New York, NY (3rd year).
- Topological quantum photonics and novel soliton physics, Andrea Blanco-Redondo (Nokia Bell Labs), at the Advanced Science Research Center, City University of New York, NY (3rd year).
- Topological physics: from photons to electrons, Mohammad Hafezi (University of Maryland), at the Advanced Science Research Center, City University of New York, NY (3rd year).
- Plasmonics on Two-Dimensional Materials, Dionisios Margetis (University of Maryland), at the Advanced Science Research Center, City University of New York, NY (3rd year).
- III-V semiconductor metasurfaces: frequency mixing and all-optical tuning, Polina Vabishchevich (Sandia National Laboratories), on-line, (3rd year).
- How to get published with the IEEE?, Eszter Lukacs (IEEE), on-line, (3rd year).
- Controlling light with metasurfaces, Franesco Monticone (Cornell University), Virtual seminar on "Metasurfaces" (3rd year).
- Bose-Einstein condensation and lasing in plasmonic lattices, Paivi Torma (Aalto university), Virtual seminar on "Metasurfaces" (3rd year).
- Vortex beams generation with dielectric metasurfaces, Antonio Ambrosio (Istituto italiano di tecnologia-CNST@poliMi), Virtual seminar on "Metasurfaces" (3rd year).
- Ultrafast phenomena workshop (online), Matthew Sfeir (Photonics Initiative, Advanced Science Research Center, CUNY, NY) (3rd year).
- 2020 CLEO Virtual Conference: Laser Science to Photonic Applications (3rd year).
- Radiative Cooling Under the Earth's Glow, Jyotirmoy Mandal (UCLA) (3rd year).
- Optical metamaterials based on broken symmetries, Andrea Alù (Photonics Initiative, Advanced Science Research Center, CUNY, NY), on-line (3rd year)
- Network Systems, Kuramoto Oscillators, and Synchronous Power Flow, Francesco Bullo, on-line (3rd year).

Publication List 1/2

Journal papers (7)

- **M. Pascale**, G. Miano, and C. Forestiere. “Spectral theory of electromagnetic scattering by a coated sphere.” In: JOSA B 34 (2017).
- C. Forestiere, G. Miano, **M. Pascale**, and R. Tricarico, “Directional scattering cancellation for an electrically large dielectric sphere.” In: Optics Letters 44.8 (Apr. 2019), pp. 1972–1975. issn: 1539-4794.
- C. Forestiere, G. Miano, **M. Pascale**, and R. Tricarico, “Electromagnetic Scattering Resonances of Quasi-1-D Nanoribbons.” In: IEEE Transactions on Antennas and Propagation 67.8 (Aug. 2019). pp. 5497–5506. issn: 1558-2221.
- C. Forestiere, G. Gravina, G. Miano, **M. Pascale**, and R. Tricarico. “Electromagnetic modes and resonances of two-dimensional bodies.” In: Phys. Rev. B 99.15 (Apr. 2019). Publisher: American Physical Society, p. 155423.
- **M. Pascale**, G. Miano, R. Tricarico, and C. Forestiere. “Full-wave electromagnetic modes and hybridization in nanoparticle dimers.” In: Scientific Reports 9.1 (Oct. 10, 2019). Number: 1 Publisher: Nature Publishing Group, p. 14524. issn: 2045-2322.
- C. Forestiere, G. Miano, G. Rubinacci, **M. Pascale**, A. Tamburrino, R. Tricarico, and S. Ventre. “Magnetoquasistatic resonances of small dielectric objects.” In: Phys. Rev. Research 2.1 (Feb. 2020), p. 013158.
- C Forestiere, G Miano, **M Pascale**, R Tricarico, Quantum theory of radiative decay rate and frequency shift of surface plasmon modes, Physical Review A 102 (4), 043704.

Publication List 2/2

Journal papers under preparation (2)

- **M. Pascale**, S. Mann, C. Forestiere, A. Alù. "On the Q factor on singular plasmonic resonators: lower bounds and relation with fractional bandwidth".
- **M. Pascale**, D. Tzarouchis, G. Miano, S. Mann, A. Alù, C. Forestiere. "Lower bounds to quality factor of small radiators through quasistatic scattering modes".

Conference papers (2)

- **M. Pascale**, R. Tricarico, G. Miano and C. Forestiere, Full wave mode hybridization in nanoparticle dimers, 2019 International Conference on Electromagnetics in Advanced Applications (ICEAA 19), Granada, Spain, 2019.
- R. Tricarico, C. Forestiere, G. Miano and **M. Pascale**, Field Quantization in Arbitrarily Shaped Metal, Nanoparticles, International Conference on Electromagnetics in Advanced Applications (ICEAA 19), Granada, Spain, 2019.

Book chapters (2)

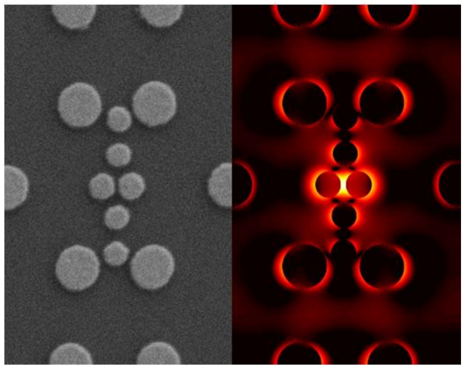
- C. Forestiere, G. Miano, **M. Pascale**, R. Tricarico, chapter title: "A full retarded spectral technique for the Fano resonance analysis in a dielectric nanosphere", Springer book: "Fano Resonances in Optics and Microwaves", pp. 185 218, Nov. 2018.
- C. Forestiere, G. Miano, **M. Pascale**, R. Tricarico , chapter title: "Material Independent Modes for the design of electromagnetic scattering", World Scientific Publishing book: "Compendium on Electromagnetic Analysis from electrostatics to photonics: fundamentals and applications for physicists and engineers", out in Apr. 2020.

Credit Summary

	Credits year 1								Credits year 2								Credits year 3								Extension					
		1	2	3	4	5	6			1	2	3	4	5	6			1	2	3	4	5	6		7	8	9			
	Estimated	bimonth	bimonth	bimonth	bimonth	bimonth	bimonth	**Summary**	**Estimated**	bimonth	bimonth	bimonth	bimonth	bimonth	bimonth	**Summary**	**Estimated**	bimonth	bimonth	bimonth	bimonth	bimonth	bimonth	**Summary**	bimonth	bimonth	month	**Summary**	**Total**	**Check**
Modules	**20**	0	0	8	0	0	2.3	**10.3**	**20**	10.6	6	6.8	0	0	0	**23.4**	**6**	0	0	0	0	0	6	**6**	0	0	0	**0**	**39.7**	**30-70**
Seminars	**5**	0	0	1.8	0	3	0.4	**5.2**	**5**	0.6	4.3	0.4	0	1.4	0.2	**6.9**	**5**	0.6	1.4	2.9	0	0	0.3	**5.2**	0	0	0	**0**	**17.3**	**10-30**
Research	**35**	7	7.5	7	7	9	7	**44.5**	**35**	1	2	3.3	8	7	8.4	**29.7**	**49**	6	5	5	9	9	3.8	**37.8**	9	9	4	**22**	**134**	**80-140**
	60	7	7.5	16.8	7	12	9.7	**60**	**60**	12.2	12.3	10.5	8	8.4	8.6	**60**	**60**	6.6	6.4	7.9	9	9	10	**49**	9	9	4	**22**	**191**	**180**

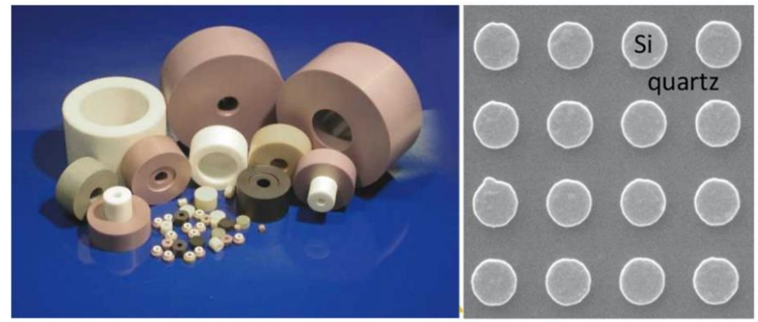
Electromagnetic resonances of electrically small objects

Metal objects



- Nano-optics
- Sensing
- Boosting non-linear and quantum effects
- Meta-atoms

Dielectric objects

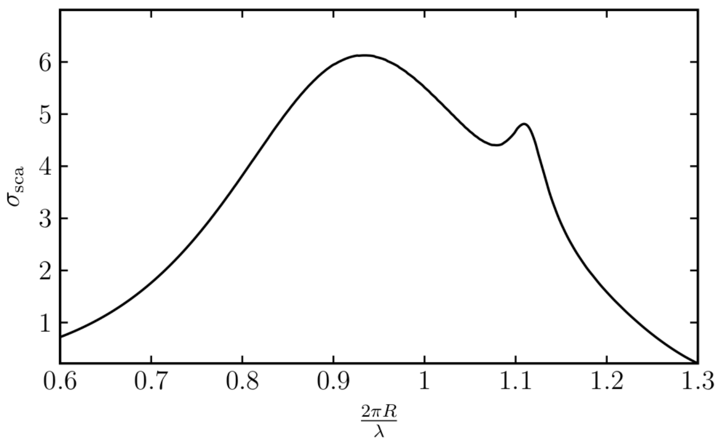


Nano Lett. 2017, 17, 7, 4421-4426

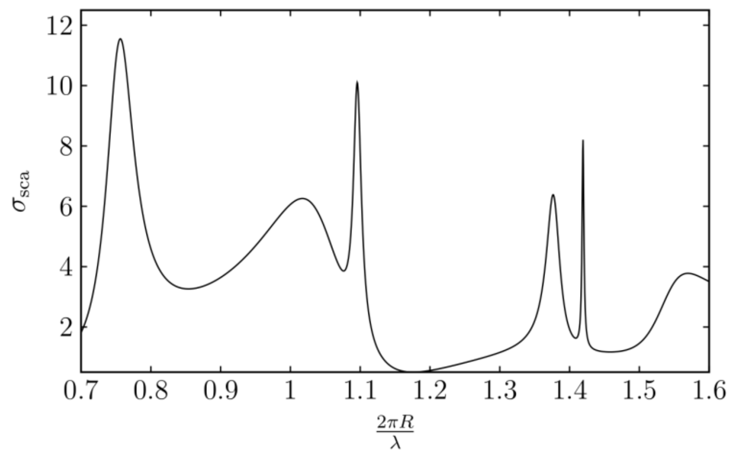
- Nano-optics and RF
- RF antennas and filters
- Resonant energy transfer
- Meta-atoms

Resonances in Metal and Dielectric spherical nanoparticles

Silver: $\varepsilon_R < 0$ R=67.5 nm



Silicon: $\varepsilon_R = 16$



Resonances in Metal and Dielectric spherical nanoparticles

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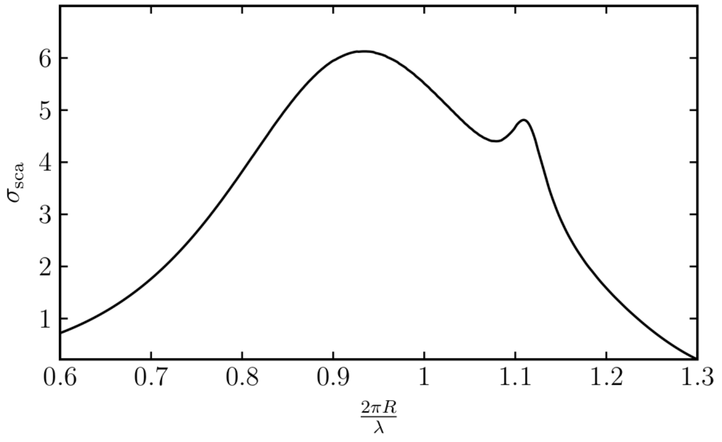
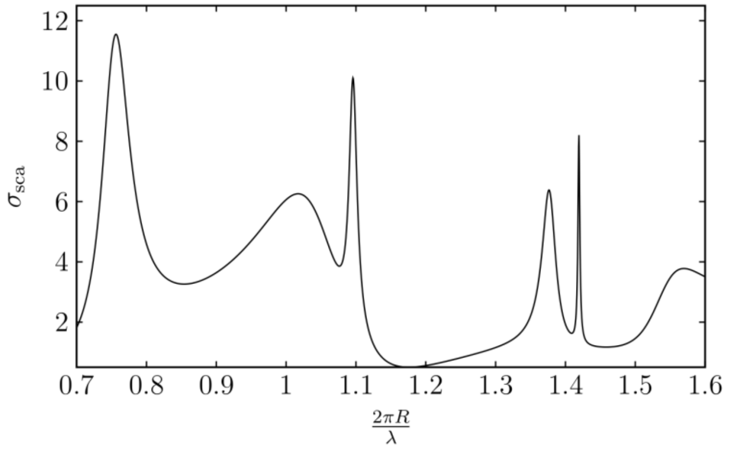
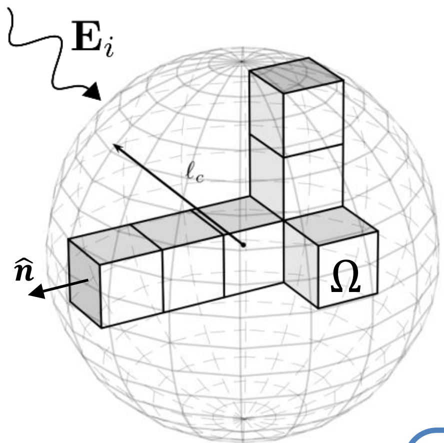


Silicon: $\varepsilon_R = 16$



- Why do dielectric and metal NSs of comparable size exhibit **deeply different behaviours?**
- **Magnetic (TE) modes** in isolated metal Ns at optical frequencies have never been seen, but they can be excited in their Si counterpart.
- Why have asymmetric lineshapes in the total scattering spectrum been observed for Si spheres but not for metal NSs?

Electromagnetic Scattering



- Homogeneous, isotropic, nonmagnetic, linear material
- Volume Ω bounded by a closed surface $\partial\Omega$ with normal $\hat{\boldsymbol{n}}$
- Characteristic linear dimension ℓ_c
- Relative dielectric permittivity ε_R, electric susceptibility $\chi = \varepsilon_R - 1$

$$\mathbf{e}_{\mathrm{inc}}(\mathrm{t}) = \mathrm{Re}\{\mathbf{E}_{\mathrm{inc}}(\mathbf{r})\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}\}$$

$$x = \frac{2\pi\ell_c}{\lambda}$$

Unknown: induced current density field **J**(**r**)

$$\mathbf{J}(\mathbf{r}) \ = \ -i\omega\varepsilon_0\chi\mathbf{E}(\mathbf{r}) \quad \forall\mathbf{r}\in\Omega \qquad\qquad \mathbf{E} = \mathbf{E}_{\mathrm{inc}} + \mathbf{E}_{\mathrm{sca}}$$

After spatial coordinate normalization: $\boldsymbol{r} \to \boldsymbol{r}/\ell_c$

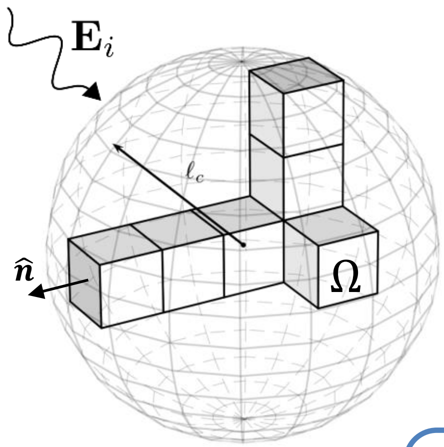
$$\frac{\mathbf{J}(\mathbf{r})}{\chi} - \mathcal{L}\{\mathbf{J}\}(\mathbf{r}) = -i\omega\varepsilon_0\,\mathbf{E}_{\mathrm{inc}}(\mathbf{r}), \quad \forall\mathbf{r}\in\Omega$$

Full-wave operator

$$\mathcal{L}\{\mathbf{W}\}(\mathbf{r}) = -\nabla\oint_{\partial\Omega}\mathbf{W}(\mathbf{r}')\cdot\hat{\mathbf{n}}'\frac{e^{ix|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}\,dS' + x^2\int_{\Omega}\mathbf{W}(\mathbf{r}')\frac{e^{ix|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}\,dV',$$

Ph.D itee INFORMATION TECHNOLOGY ELECTRICAL ENGINEERING

Electromagnetic Scattering



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Full-wave operator

$$\mathcal{L}\{\mathbf{W}\}(\mathbf{r}) = -\nabla\oint_{\partial\Omega}\mathbf{W}(\mathbf{r}')\cdot\hat{\mathbf{n}}'\frac{e^{ix|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}\,dS' + x^2\int_{\Omega}\mathbf{W}(\mathbf{r}')\frac{e^{ix|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}\,dV',$$

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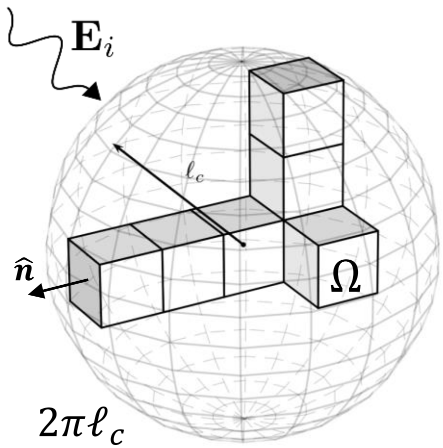
HOW TO SOLVE IT?

Spectral theory



$$x = \frac{2\pi\ell_c}{\lambda}$$

Unknown: induced current density field **J**(**r**)

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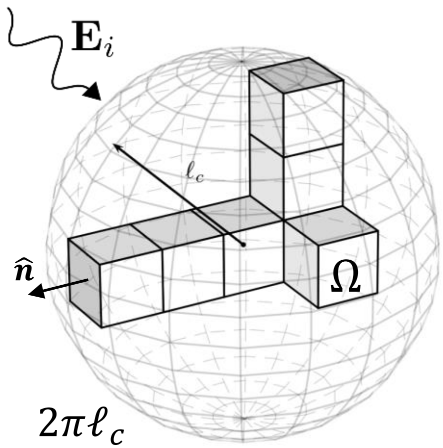
$$\frac{\mathbf{J}(\mathbf{r})}{\chi} - \mathcal{L}\{\mathbf{J}\}(\mathbf{r}) = -i\omega\varepsilon_0\mathbf{E}_{\text{inc}}(\mathbf{r}), \quad \forall \mathbf{r} \in \Omega$$

$$\mathcal{L}\{\mathbf{W}\}(\mathbf{r}) = -\nabla \oint_{\partial\Omega} \mathbf{W}(\mathbf{r}') \cdot \hat{\mathbf{n}}' \frac{e^{ix|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} dS' + x^2 \int_{\Omega} \mathbf{W}(\mathbf{r}') \frac{e^{ix|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} dV'$$

We do not solve the problem *directly*, but we provide a modal theory for the scattering in open systems:

- offers intuitive insights into the physics of the problem;
- enables the rigorous comprehension of interference phenomena, including Fano resonances, as the interplay among well-identified modes;
- suggests how to shape the excitation (or the material) to achieve a prescribed tailoring of the scattering response.

Spectral theory



$$x = \frac{2\pi\ell_c}{\lambda}$$

Unknown: induced current density field **J**(**r**)

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Linear algebra analogy

$$Ax = b$$
$$A = A^+$$

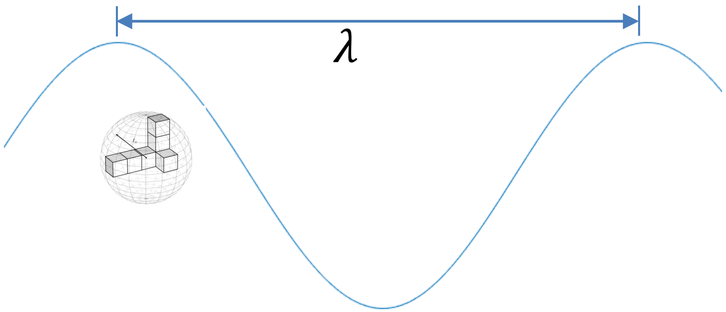
Direct solution

$$x = A^{-1}b$$

Modal solution

$$Ac_i = \lambda_i c_i \qquad x = \sum_i \frac{1}{\lambda_i}\frac{\langle c_i, b\rangle}{\langle c_i, c_i\rangle}c_i$$

Quasistatic regime



$$\ell_c \ll \lambda$$

$$x = \frac{2\pi \ell_c}{\lambda} \ll 1$$

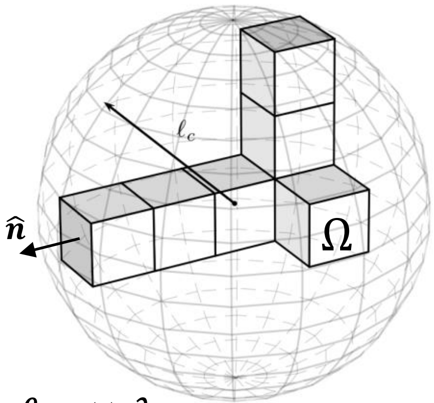
No radiation of energy to infinity (<u>closed system</u>)

Helmholtz decomposition for the solenoidal induced vector field **J:**

$$\mathbf{J} = \mathbf{J}^{\parallel} + \mathbf{J}^{\perp}$$

- $\nabla \times \mathbf{J}^{\parallel} = \mathbf{0},\ \mathbf{J}^{\parallel} \cdot \hat{\boldsymbol{n}} \neq 0$ →**Longitudinal** component of **J**
- $\nabla \times \mathbf{J}^{\perp} \neq \mathbf{0},\ \mathbf{J}^{\perp} \cdot \hat{\boldsymbol{n}} = 0$ →**Transverse** component of **J**

Electroquasistatic (EQS) resonances



$\ell_c \ll \lambda$

$$x = \frac{2\pi\ell_c}{\lambda} \ll 1$$

$\nabla \times \mathbf{J}^{\parallel} = \mathbf{0},\ \mathbf{J}^{\parallel} \cdot \hat{\boldsymbol{n}} \neq 0$ →**Longitudinal** component of **J**

It can be expanded in electroquasistatic modes:

$$\mathbf{J}^{\parallel} = \sum_h \alpha_h \, \mathbf{j}_h^{\parallel}$$

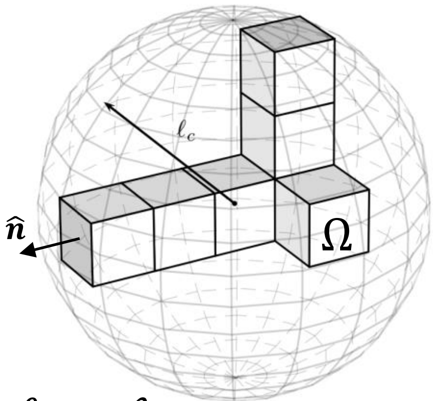
$$\mathcal{L}_e\left\{\mathbf{j}_h^{\parallel}\right\}(\mathbf{r}) = \frac{1}{\chi_h^{\parallel}} \mathbf{j}_h^{\parallel}(\mathbf{r}), \quad \forall \mathbf{r} \in \Omega$$

Electroquasistatic integral operator

$$\mathcal{L}_e\{\mathbf{W}\} = -\nabla \oint_{\partial\Omega} \frac{\mathbf{W}(\mathbf{r}') \cdot \hat{\mathbf{n}}'}{4\pi|\mathbf{r}-\mathbf{r}'|} dS'$$

$\mathbf{j}_h^{\parallel}$ describe the plasmon resonances in small metal nanoparticles with negative permittivity, arising from the interplay between the energy stored in the electric field and the kinetic energy of the free electrons of the metal.

Electroquasistatic (EQS) resonances



$\ell_c \ll \lambda$

$$x = \frac{2\pi\ell_c}{\lambda} \ll 1$$

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$$\mathcal{L}_e\{\mathbf{W}\} = -\nabla \oint_{\partial\Omega} \frac{\mathbf{W}(\mathbf{r}')\cdot\hat{\mathbf{n}}'}{4\pi|\mathbf{r}-\mathbf{r}'|}\,dS'$$

- The spectrum of $\mathcal{L}_e$ is discrete
- The EQS eigenvalues $\chi_h^{\parallel} \in \mathbb{R}^-$ and size-independent
- The EQS modes $\mathbf{j}_h^{\parallel}$ are longitudinal vector fields and orthonormal:

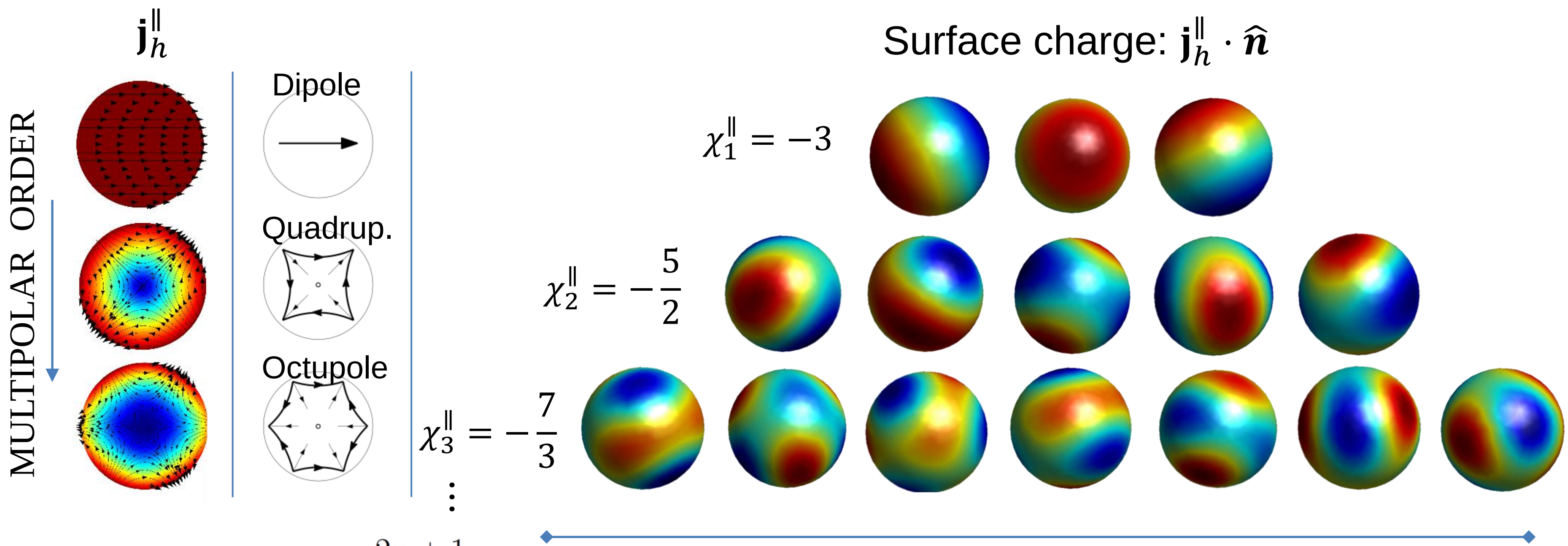
$$\langle \mathbf{j}_h^{\parallel}, \mathbf{j}_k^{\parallel} \rangle_\Omega = \delta_{h,k} \qquad \langle \mathbf{A}, \mathbf{B} \rangle_\Omega = \int_\Omega \mathbf{A}\cdot\mathbf{B}\,dV$$

- The electrostatic energy of the h-th mode is

$$\mathscr{W}_e\left\{\mathbf{j}_h^{\parallel}\right\} = \frac{1}{2\varepsilon_0}\frac{1}{\left(-\chi_h^{\parallel}\right)}$$

- The EQS resonance frequency $\omega_h^{\parallel}$ of the h-th EQS mode $\mathbf{j}_h^{\parallel}$ is defined as the frequency at which: $\mathrm{Re}\left\{\chi(\omega_h^{\parallel})\right\} = \chi_h^{\parallel}$
- Their electric dipole moment is defined as: $\mathbf{P}_h = \int_\Omega \mathbf{j}_h^{\parallel} dV$

EQS resonances: sphere



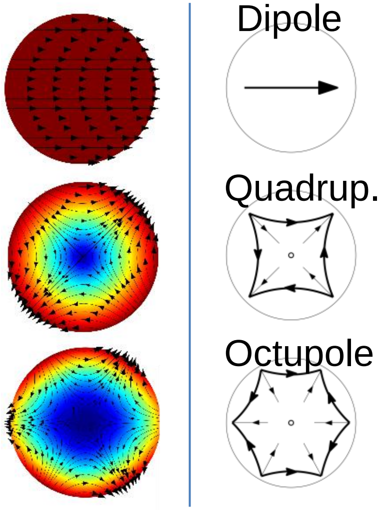
$$\chi_n^{\parallel} = -\frac{2n+1}{n}$$

Mariano Pascale

EQS resonances: sphere

$\mathbf{j}_h^{\parallel}$

MULTIPOLAR ORDER



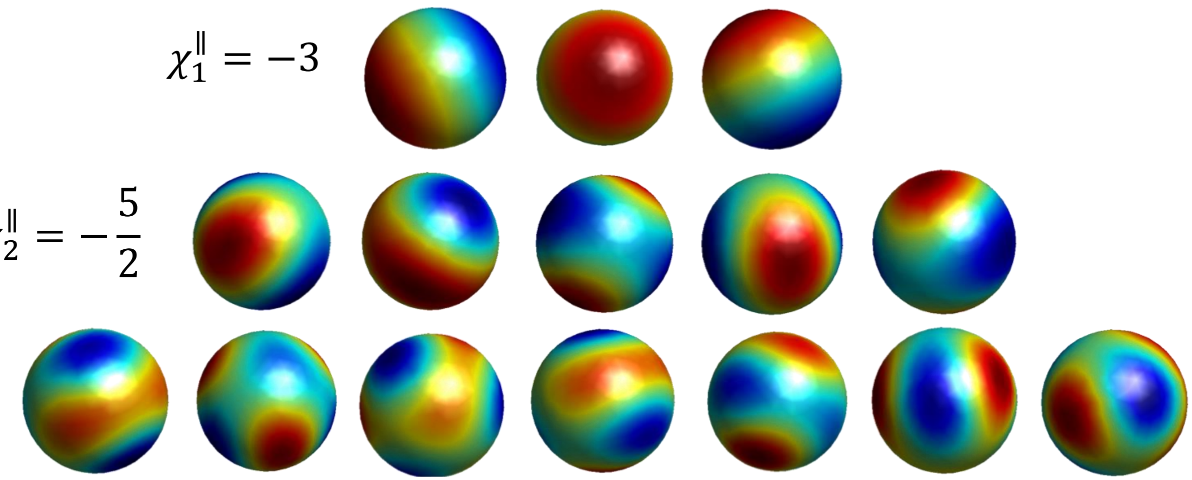
Surface charge: $\mathbf{j}_h^{\parallel} \cdot \hat{\boldsymbol{n}}$

$\chi_1^{\parallel} = -3$

$\chi_2^{\parallel} = -\frac{5}{2}$

$\chi_3^{\parallel} = -\frac{7}{3}$

$\vdots$

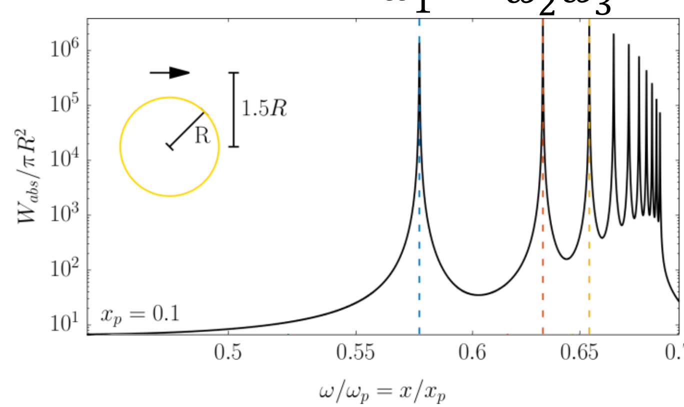


$$\chi_n^{\parallel} = -\frac{2n+1}{n}$$

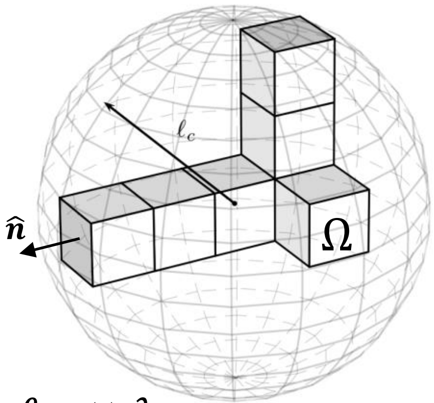
$\omega_1^{\parallel}$ $\omega_2^{\parallel} \omega_3^{\parallel} \cdots$

Drude metal

$$\varepsilon_R = 1 - \frac{\omega_p^2}{\omega(\omega + i\,\nu)}$$



Magnetoquasistatic (MQS) resonances



$\ell_c \ll \lambda$

$x = \frac{2\pi\ell_c}{\lambda} \ll 1$

$\nabla \times \mathbf{J}^{\parallel} \neq \mathbf{0},\ \mathbf{J}^{\parallel} \cdot \hat{\boldsymbol{n}} = 0$ →**Transverse** component of **J**

It can be expanded in magnetoquasistatic modes:

$$\mathbf{J}^{\perp} = \sum_h \beta_h \, \mathbf{j}_h^{\perp}$$

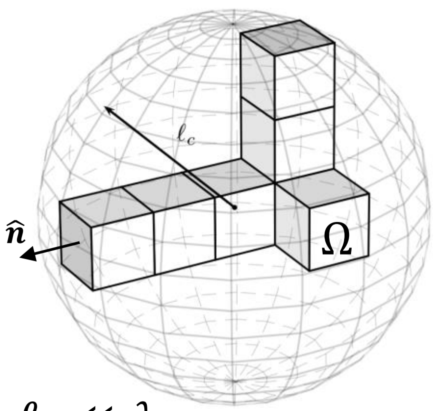
$$\mathcal{L}_m\left\{\mathbf{j}_h^{\perp}\right\}(\mathbf{r}) = \frac{1}{\kappa_h^{\perp}} \mathbf{j}_h^{\perp}(\mathbf{r}), \quad \forall \mathbf{r} \in \Omega$$

Magnetoquasistatic integral operator

$$\mathcal{L}_m\{\mathbf{W}\}(\mathbf{r}) = \int_\Omega \frac{\mathbf{W}(\mathbf{r}')}{4\pi|\mathbf{r}-\mathbf{r}'|} dV'$$

$\mathbf{j}_h^{\perp}$ describe the resonances in small objects of positive and high permittivity, arising from the interplay between the polarization energy stored in the dielectric and the energy stored in the magnetic field.

Magnetoquasistatic (MQS) resonances



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$$\mathcal{L}_m\{\mathbf{W}\}(\mathbf{r}) = \int_{\Omega} \frac{\mathbf{W}(\mathbf{r}')}{4\pi|\mathbf{r} - \mathbf{r}'|} \, dV'$$

- The spectrum of $\mathcal{L}_m$ is discrete
- The eigenvalues $\kappa_h^{\perp} \in \mathbb{R}^+$
- The modes $\mathbf{j}_h^{\perp}$ are transverse vector fields and orthonormal:

$$\langle \mathbf{j}_h^{\perp}, \mathbf{j}_k^{\perp} \rangle_{\Omega} = \delta_{h,k} \qquad \langle \mathbf{A}, \mathbf{B} \rangle_{\Omega} = \int_{\Omega} \mathbf{A} \cdot \mathbf{B} \, dV.$$

- The magnetostatic energy of the h-th mode is

$$\mathscr{W}_m \left\{ \mathbf{j}_h^{\parallel} \right\} = \frac{\mu_0}{2} \frac{1}{\kappa_h^{\perp}}$$

- The MQS resonance frequency $\omega_h^{\perp}$ of the h-th MQS mode $\mathbf{j}_h^{\perp}$ is defined as the frequency at which:

$$x_h^{\perp} = \frac{\omega_h^{\perp}}{c_0} \ell_c = \sqrt{\frac{\kappa_h^{\perp}}{\operatorname{Re}\{\chi\}}}$$

- Their magnetic dipole moment is defined as:

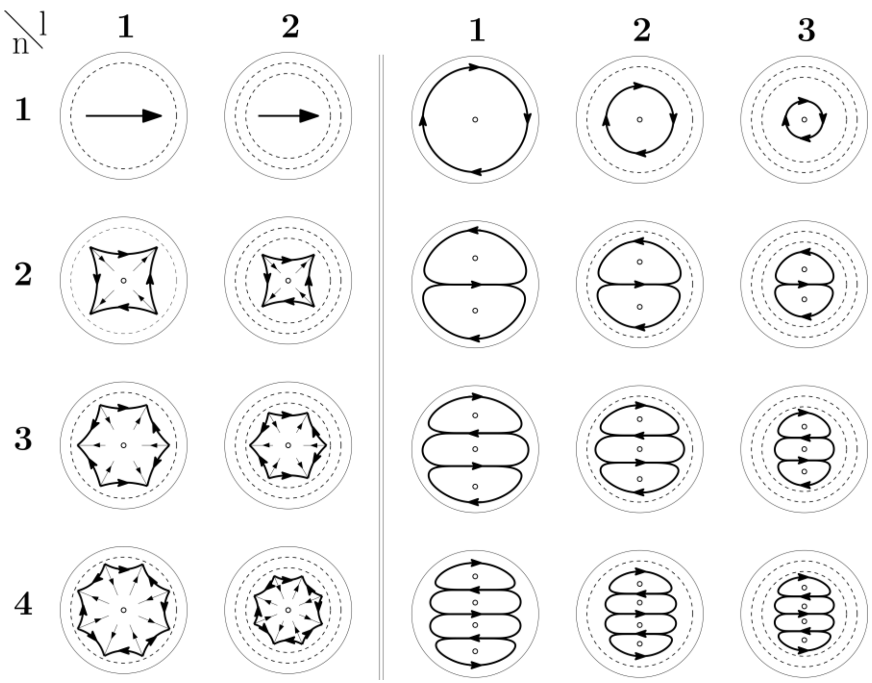
$$\mathbf{M}_h = \frac{1}{2} \int_{\Omega} \mathbf{r} \times \mathbf{j}_h^{\perp} \, dV.$$

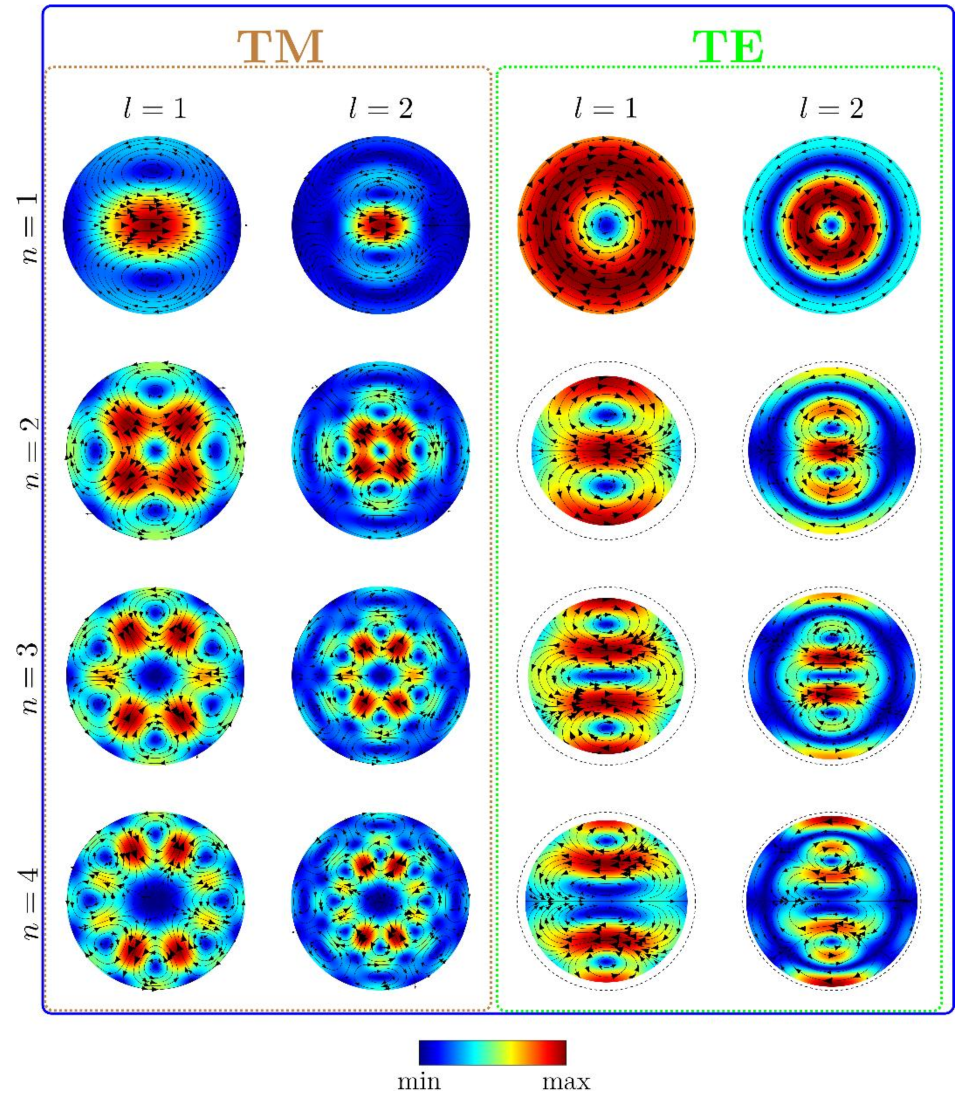
ITEE Ph.D — Information Technology Electrical Engineering

MQS resonances: sphere

- $\kappa_{nl}^{\perp\,\mathrm{TM}} = (z_{n,l})^2$;
- $\kappa_{nl}^{\perp\,\mathrm{TE}} = (z_{n-1,l})^2$

$z_{n,l}$ = l-th zero of the n-th order spherical Bessel j_n





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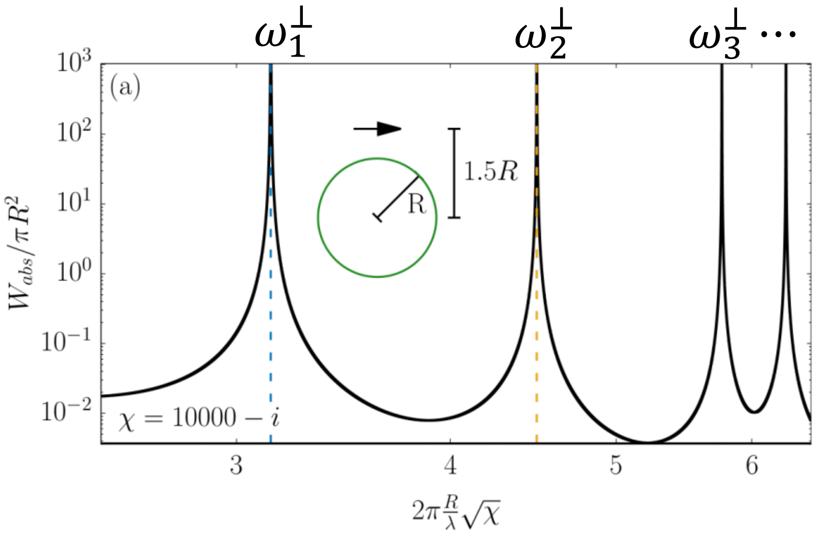
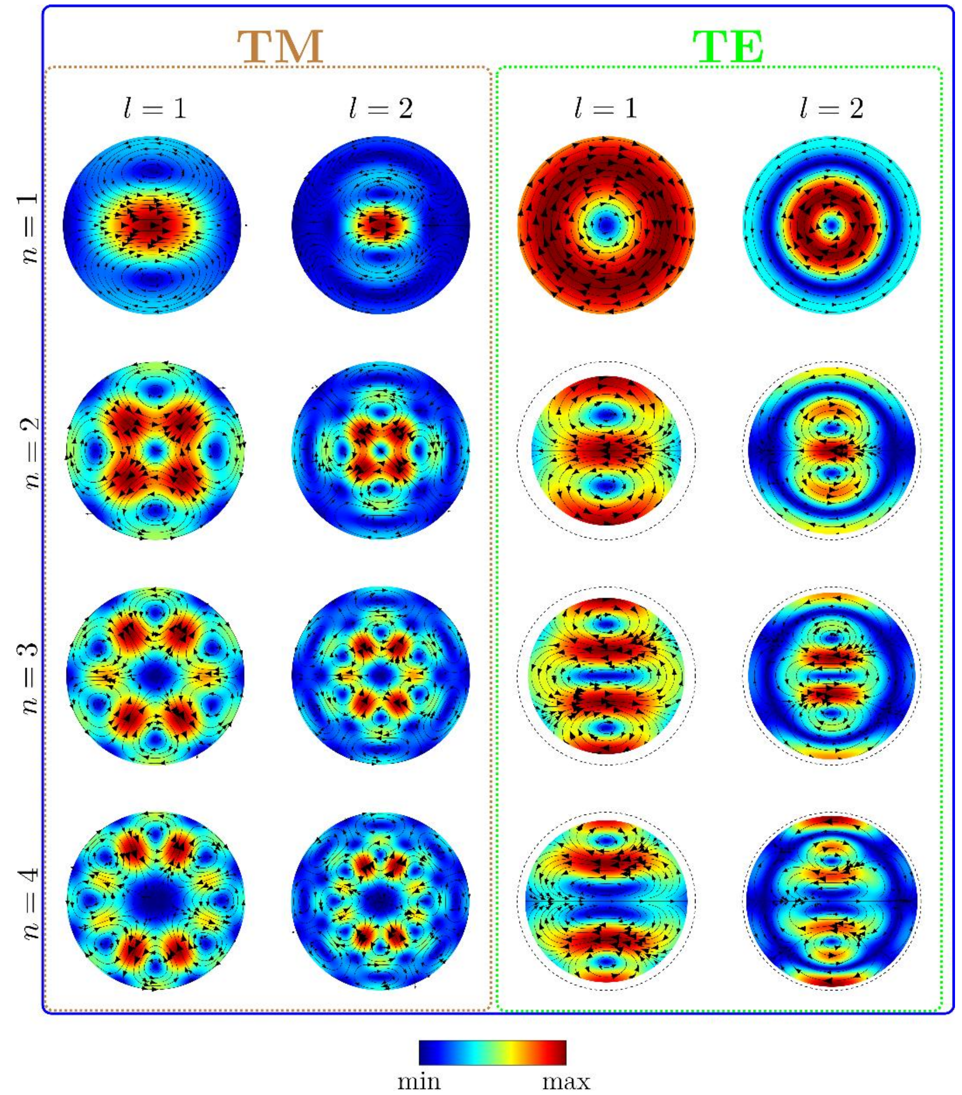
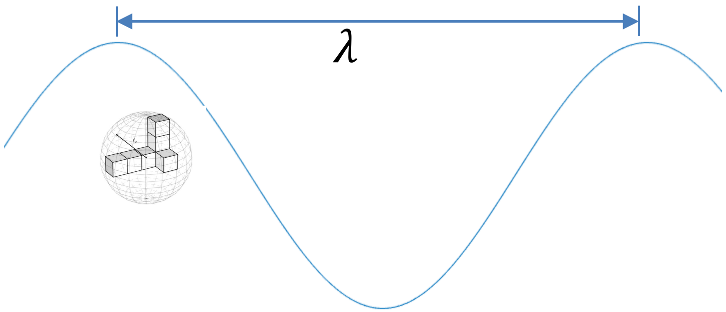




Mariano Pascale

Quasistatic Material-independent Mode (MIM) Expansion



$$\ell_c \ll \lambda$$

$$x = \frac{2\pi \ell_c}{\lambda} \ll 1$$

No radiation of energy to infinity (closed system)

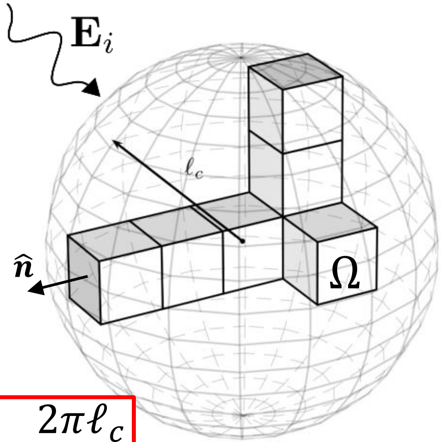
$$\mathbf{J} = -i\omega\varepsilon_0\chi \sum_{h=1}^{\infty} \left[\frac{\chi_h^{\|}}{\chi_h^{\|} - \chi} \langle \mathbf{j}_h^{\|}, \mathbf{E}_{\text{inc}} \rangle_\Omega \, \mathbf{j}_h^{\|} + \frac{\kappa_h^{\perp}}{\kappa_h^{\perp} - x^2\chi} \langle \mathbf{j}_h^{\perp}, \mathbf{E}_{\text{inc}} \rangle_\Omega \, \mathbf{j}_h^{\perp} \right]$$

EQS contribution **MQS contribution**

Resonance conditions →

$$\mathrm{Re}\{\chi\} = \chi_h^{\|} < 0 \qquad \mathrm{Re}\{\chi\} = \frac{\kappa_h^{\perp}}{x^2} \gg 1$$

Beyond the Quasistatic limit



$$x = \frac{2\pi\ell_c}{\lambda}$$
$$\ell_c \sim \lambda$$

Radiation of energy to infinity (open system)

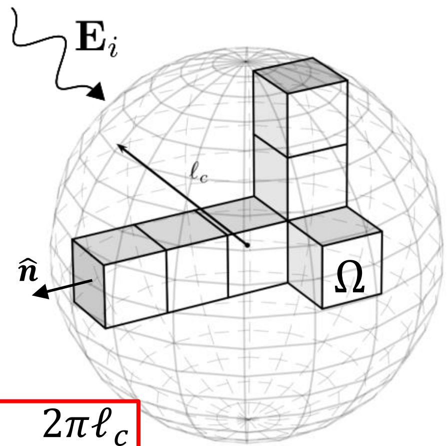
Unknown: induced current density field **J**(**r**)

$$\mathbf{J}(\mathbf{r}) = -i\omega\varepsilon_0\chi\mathbf{E}(\mathbf{r}) \quad \forall\mathbf{r}\in\Omega \qquad \mathbf{E} = \mathbf{E}_{\mathrm{inc}} + \mathbf{E}_{\mathrm{sca}}$$

$$\frac{\mathbf{J}(\mathbf{r})}{\chi} - \mathcal{L}\left\{\mathbf{J}\right\}(\mathbf{r}) = -i\omega\varepsilon_0\,\mathbf{E}_{\mathrm{inc}}(\mathbf{r}), \quad \forall\mathbf{r}\in\Omega$$

$$\mathcal{L}\left\{\mathbf{W}\right\}(\mathbf{r}) = -\nabla\oint_{\partial\Omega}\mathbf{W}(\mathbf{r}')\cdot\hat{\mathbf{n}}'\frac{e^{ix|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}\,dS' + x^2\int_{\Omega}\mathbf{W}(\mathbf{r}')\frac{e^{ix|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}\,dV'$$

Beyond the Quasistatic limit



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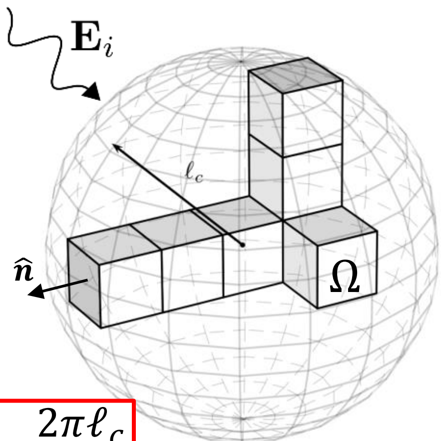
Auxiliary (full-wave) eigenvalue problem

$$\mathcal{L}\{\mathbf{j}_h\} = \frac{1}{\gamma_h}\mathbf{j}_h$$

- $\mathcal{L}$ is not self-adjoint
- Eigenvalues $\gamma_h \in \mathbb{C}$ with $\Im m\{\gamma_h\} < 0$
- Eigemodes bi-orthogonal
- For $x \to 0$ the problem splits in the two EQS and MQS eigenvalue problems

Full-wave MIM Expansion



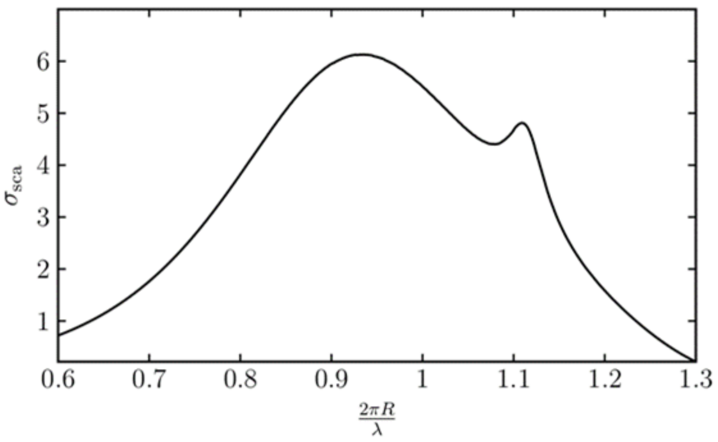
$$x = \frac{2\pi\ell_c}{\lambda}$$
$$\ell_c \sim \lambda$$

$$\mathbf{J} = -i\omega\varepsilon_0\chi\sum_{h=1}^{\infty}\left[\frac{\chi_h}{\chi_h-\chi}\langle\mathbf{u}_h,\mathbf{E}_{inc}\rangle_\Omega\,\mathbf{u}_h + \frac{\kappa_h}{\kappa_h-\chi x^2}\langle\mathbf{v}_h,\mathbf{E}_{inc}\rangle_\Omega\,\mathbf{v}_h\right]$$

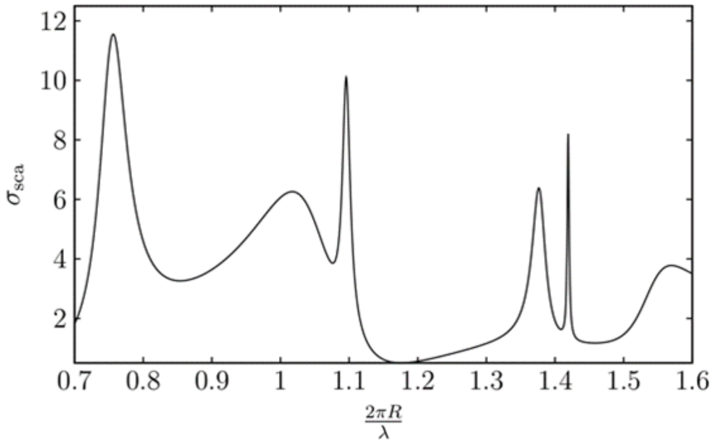
	Plasmonic modes	**Dielectric modes**
$\lim_{x\to 0}$	$\chi_h(x)\to\chi_h^{\parallel}$ $\boldsymbol{u}_h(x)\to\boldsymbol{j}_h^{\parallel}$	$\kappa_h(x)\to\kappa_h^{\perp}$ $\boldsymbol{v}_h(x)\to\boldsymbol{j}_h^{\perp}$
Resonance conditions	$\min_x\left\lvert\frac{\chi_h(x)-\chi(x)}{\chi(x)}\right\rvert=0$	$\min_x\left\lvert\frac{\kappa_h(x)/x^2-\chi(x)}{\chi(x)}\right\rvert=0$

Resonances in Metal and Dielectric spherical nanoparticles

Silver: $\varepsilon_R < 0$ R=67.5 nm



Silicon: $\varepsilon_R = 16$



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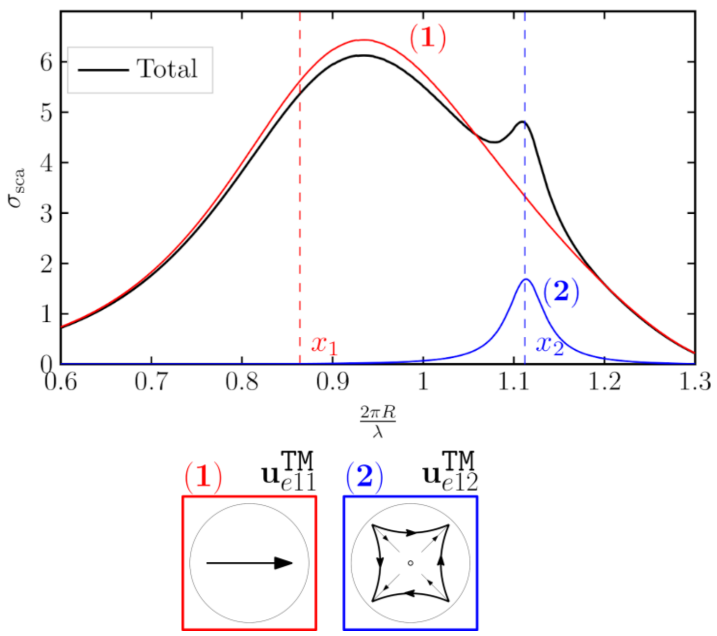

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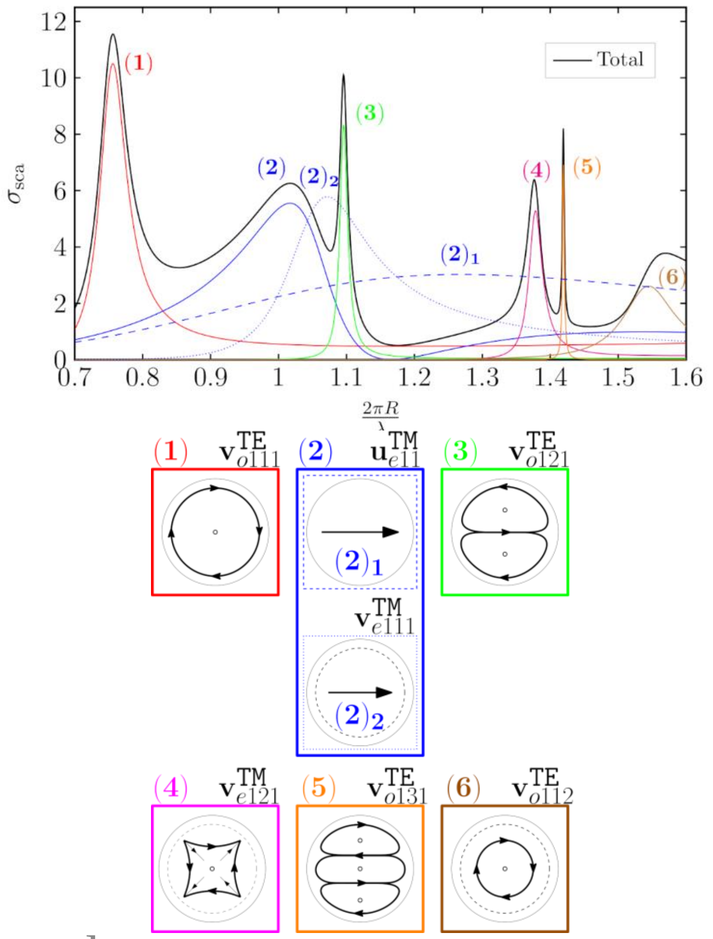
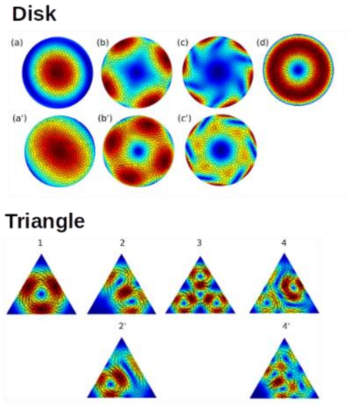
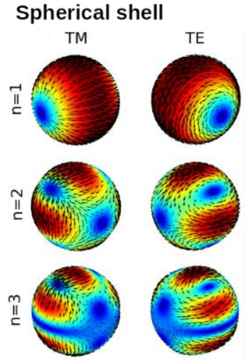


MIMs: some works

2D STRUCTURES

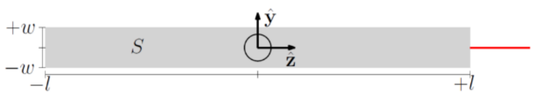
Full-wave modes and resonances of arbitrary shaped surfaces.





C. Forestiere, G. Gravina, G. Miano, M. Pascale, and R. Tricarico Phys. Rev. B 99, 155423

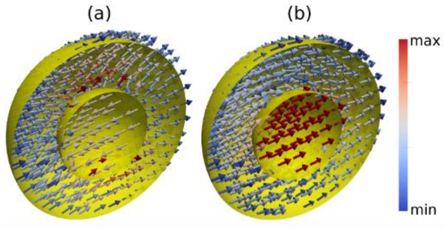
QUASI 1-D NANORIBBONS



C. Forestiere, G. Miano, M. Pascale, and R. Tricarico, "Electromagnetic Scattering Resonances of Quasi-1-D Nanoribbons." In: IEEE Transactions on Antennas and Propagation 67.8 (Aug. 2019). pp. 5497–5506. issn: 1558-2221.

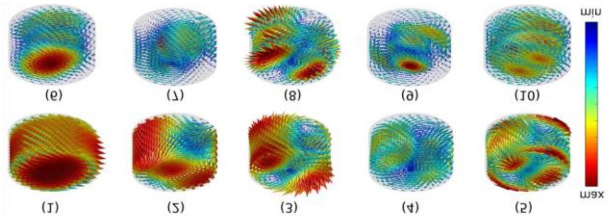
ELECTROMAGNETIC CLOAKING

Full-wave modes and resonances of a homogeneusly coated sphere: field maximization and directional scattering cancellation.



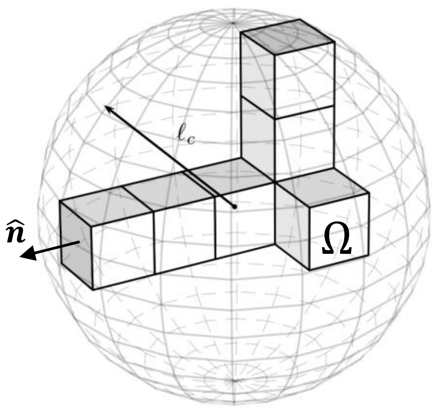
M. Pascale, G. Miano, and C. Forestiere. "Spectral theory of electromagnetic scattering by a coated sphere." In: JOSA B 34 (2017).
C. Forestiere, G. Miano, M. Pascale, and R. Tricarico, "Directional scattering cancellation for an electrically large dielectric sphere." In: Optics Letters 44.8 (Apr. 2019), pp. 1972–1975. issn: 1539-4794.

ARBITRARY SHAPED 3D STRUCTURES



C. Forestiere, G. Miano, G. Rubinacci, A. Tamburrino, R. Tricarico, and S. Ventre. "Volume Integral Formulation for the Calculation of Material Independent Modes of Dielectric Scatterers." In: *IEEE Transactions on Antennas and Propagation* 66.5 (May 2018), pp. 2505–2514.

Resonances in Electrically Small objects



$$x = \frac{2\pi\ell_c}{\lambda} < 1$$

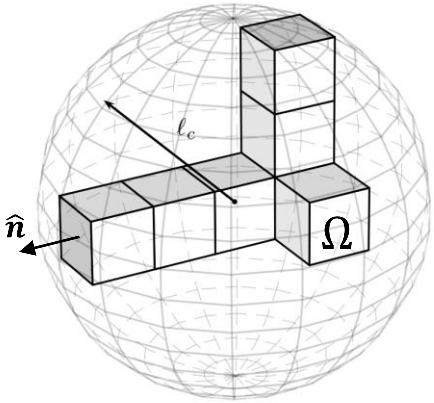
Perturbation approach:

- The full-wave eigenvalue problem is solved perturbatively starting from the EQS and MQS limits
- The size parameter *x* is treated as small parameter

$$\kappa_h = \kappa_h^{\perp} + \kappa_h^{(1)} x + \kappa_h^{(2)} x^2 + \ldots$$

$$\chi_h = \chi_h^{\parallel} + \chi_h^{(1)} x + \chi_h^{(2)} x^2 + \ldots$$

Resonances in Electrically Small objects: Q factor



$$x = \frac{2\pi \ell_c}{\lambda} < 1$$

Resonance Radiation Quality (Q) factor:

- Low Material losses: $Im\{\varepsilon_R\} \ll Re\{\varepsilon_R\}$
- Ratio between the maximum of the stored electric and magnetic energies, and the radiated power toward infinity
- High Q = inverse of the fractional bandwidth

Mode radiation Q factor

Plasmonic modes

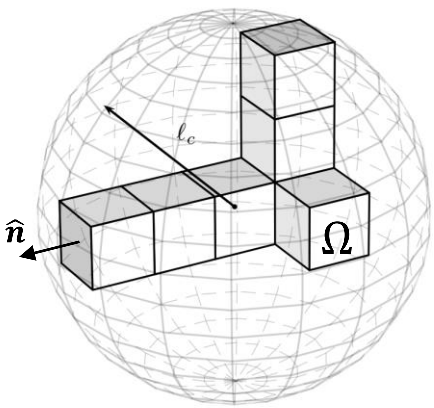
$$Q_h^{\parallel} = \left| \frac{\chi_h^{\parallel}}{\chi_h^{(n_i)}} \right| \left(\frac{1}{x_h} \right)^{n_i}$$

Dielectric modes

$$Q_h^{\perp} = \left| \frac{\kappa_h^{\perp}}{\kappa_h^{(n_i)}} \right| \left(\frac{1}{x_h} \right)^{n_i}$$

- n_i= order of the first nonzero imaginary correction

Resonances in Electrically Small objects: Q factor



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Mode radiation Q factor

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$$Q_h^{||} = \left|\frac{\chi_h^{||}}{\chi_h^{(n_i)}}\right| \left(\frac{1}{x_h}\right)^{n_i}$$

$\mathbf{P}_h \neq \mathbf{0}$

$$Q_h^{||} = \frac{6\pi}{(-\chi_h^{||})|\mathbf{P}_h|^2} \left(\frac{1}{x_h}\right)^3$$

Dielectric modes

$$Q_h^{\perp} = \left|\frac{\kappa_h^{\perp}}{\kappa_h^{(n_i)}}\right| \left(\frac{1}{x_h}\right)^{n_i}$$

$\mathbf{M}_h \neq \mathbf{0}$

$$Q_h^{\perp} = \frac{6\pi}{\kappa_h^{\perp}|\mathbf{M}_h|^2} \left(\frac{1}{x_h}\right)^3$$

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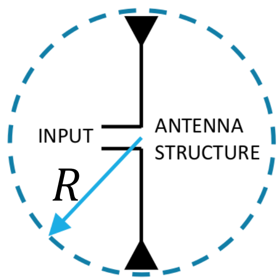
Q factor: Chu-limit

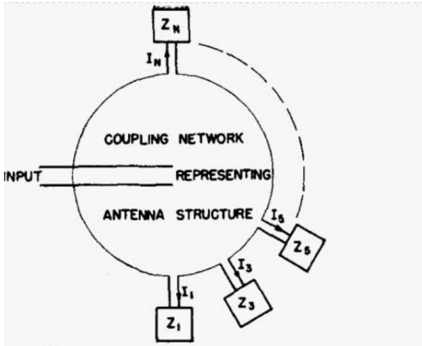
Vertically polarized omni-directional antenna



Field component outside the sphere

$$\mathrm{E} = \sum_{n=1}^{\infty} a_n TM_n + b_n TE_n$$

TE, TM → Spherical waves



. Equivalent circuit of a vertically polarized omni-directional antenna.

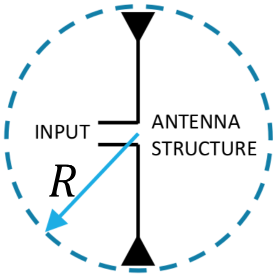
$$TM_n \rightarrow Q_n = \frac{2\omega W_n}{P_n} = \frac{1}{Bandwidth}$$

- If matched externally with stored magnetic energy
- $Q_n \gg 1$
- W_n →average stored electric energy
- P_n →average power dissipation
- $Q_{tot} = q_n Q_n, \quad q_n > 0$

Chu, L. J. (December 1948). "Physical limitations of omni-directional antennas" *Journal of Applied Physics*. **19** (12): 1163–1175

Q factor: Chu-limit

Vertically polarized omni-directional antenna



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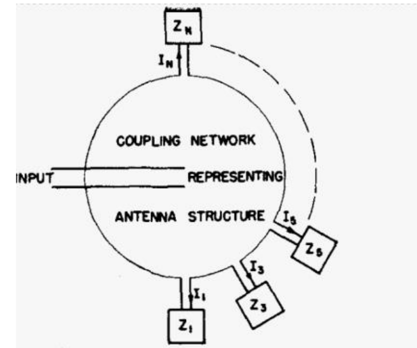

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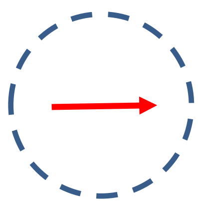
- P_n →average power dissipation
- $Q_{tot} = q_n Q_n, \quad q_n > 0$

Chu-limit

$$Q_{tot} > \frac{1}{x^3}, \qquad x = \frac{2\pi R}{\lambda}$$

Antennas enclosed in small volumes tend to have:

- High Q
- Narrow bandwidth

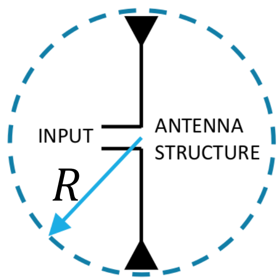


Broadest bandwidth

Chu, L. J. (December 1948). "Physical limitations of omni-directional antennas" *Journal of Applied Physics*. **19** (12): 1163–1175

Q factor: Chu-limit

Vertically polarized omni-directional antenna



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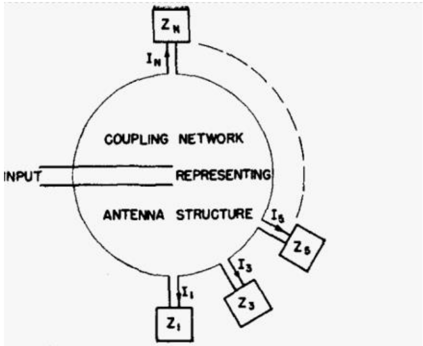

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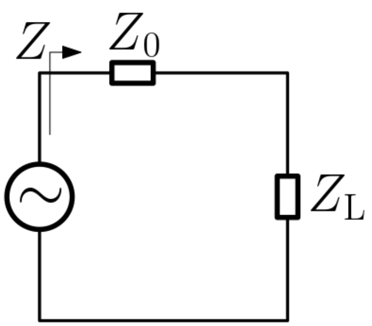
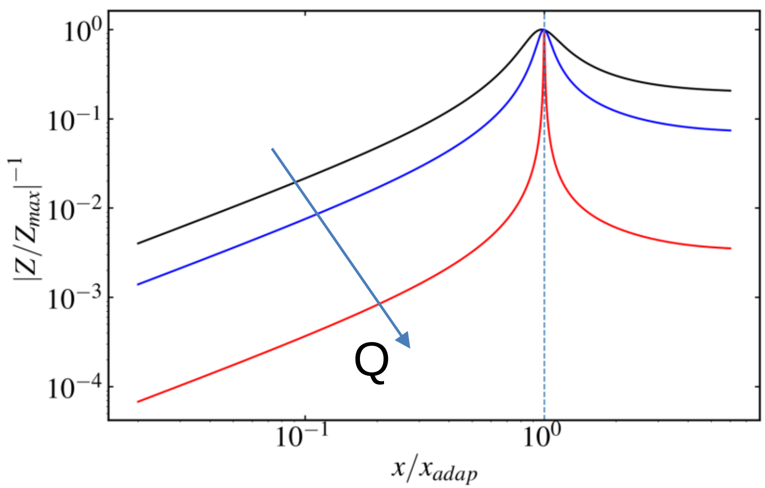


$$\mathrm{Z}_0(x_{adap}) = Z_L^*(x_{adap})$$



$$Z_n = \mathrm{i}\frac{(x\,h_n(x))'}{x\,h_n(x)}$$

hn=spherical hankel

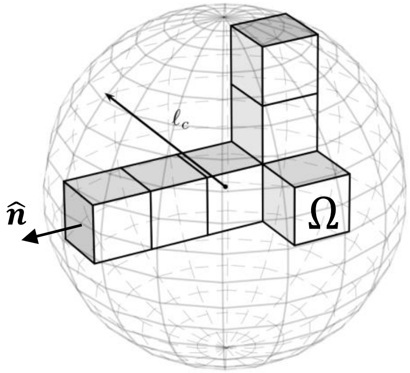
$\mathrm{Z_L = Z_1}$ DIPOLE

$\mathrm{Z_L = Z_2}$ QUADRUPOLE

$\mathrm{Z_L = Z_1 + Z_2}$ DIPOLE + QUADRUPOLE

Chu, L. J. (December 1948). "Physical limitations of omni-directional antennas" *Journal of Applied Physics*. **19** (12): 1163–1175

Shape Dependent Q factor



$$x = \frac{2\pi\ell_c}{\lambda} < 1$$

Bounds stricter than the Chu-limit:

- For any shape there exists an *optimal* current distribution yelding the minimum supported Q-factor, always greater than the Chu-limit

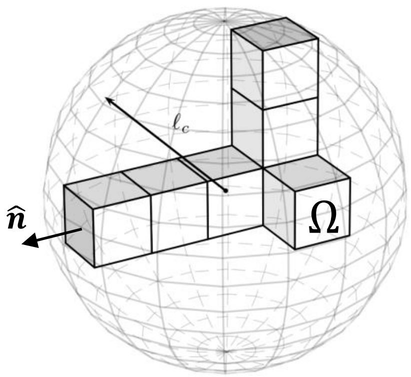
Electric type radiators:
Supporting longitudinal currents
$\nabla \times \mathbf{J} = \mathbf{0}, \mathbf{J} \cdot \hat{\boldsymbol{n}} \neq 0$

EXACTLY LIKE THE **EQS** MODES!

Magnetic type radiators
Supporting transverse currents
$\nabla \times \mathbf{J} \neq \mathbf{0},\ \mathbf{J} \cdot \hat{\boldsymbol{n}} = 0$

EXACTLY LIKE THE **MQS** MODES!

Shape Dependent Q factor



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Electric type radiators:

Supporting <u>longitudinal</u> currents

$\nabla \times \mathbf{J} = \mathbf{0}, \mathbf{J} \cdot \hat{\boldsymbol{n}} \neq 0$

EXACTLY LIKE THE **EQS** MODES!

$$x^3 Q_{min} = \frac{6\pi}{\gamma_{e,max}}$$

$\gamma_{e,max} = max$ eigenvalue of the
Electric Polarizability tensor $\boldsymbol{\gamma}_e$:

a linear correspondence between an homogeneous external displacement field $\hat{\boldsymbol{e}}\varepsilon_0 E_0$ and the electric dipole moment **P:**

$$\boldsymbol{\gamma}_e \cdot \hat{\boldsymbol{e}}\varepsilon_0 E_0 = \boldsymbol{P}$$

Magnetic type radiators

Supporting <u>transverse</u> currents

$\nabla \times \mathbf{J} \neq \mathbf{0},\ \mathbf{J} \cdot \hat{\boldsymbol{n}} = 0$

EXACTLY LIKE THE **MQS** MODES!

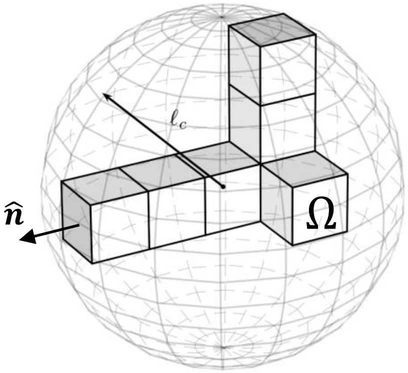
$$x^3 Q_{min} = \frac{6\pi}{\gamma_{m,max}}$$

$\gamma_{m,max} = max$ eigenvalue of the
Magnetic Polarizability tensor $\boldsymbol{\gamma}_m$:

a linear correspondence between an homogeneous external magnetic field $\hat{\boldsymbol{e}} H_0$ and the magnetic dipole moment **M**

$$\boldsymbol{\gamma}_m \cdot \hat{\boldsymbol{e}} H_0 = \boldsymbol{M}$$

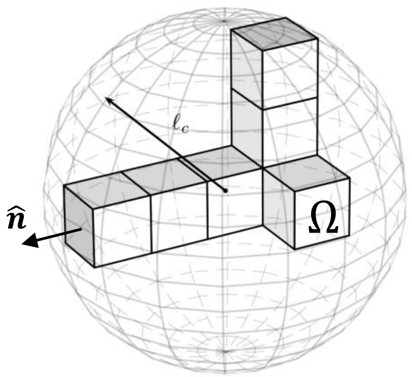
Shape Dependent Q factor through Quasistatic MIMs



$$x = \frac{2\pi\ell_c}{\lambda} < 1$$

Electric type radiators: Supporting longitudinal currents $\nabla \times \mathbf{J} = \mathbf{0}, \mathbf{J} \cdot \hat{\boldsymbol{n}} \neq 0$	**Magnetic type radiators** Supporting transverse currents $\nabla \times \mathbf{J} \neq \mathbf{0},\ \mathbf{J} \cdot \hat{\boldsymbol{n}} = 0$
$\mathbf{j}_{opt} = \sum_{h=1}^{\infty} a_h\, \mathbf{j}_h^{\parallel}$ $\boldsymbol{j}_h^{\parallel} \rightarrow$ EQS modes	$\mathbf{j}_{opt} = \sum_{h=1}^{\infty} b_h\, \mathbf{j}_h^{\perp}$ $\boldsymbol{j}_h^{\perp} \rightarrow$ MQS modes

Shape Dependent Q factor through Quasistatic MIMs



$$x = \frac{2\pi\ell_c}{\lambda} < 1$$

	Electric type radiators: Supporting longitudinal currents $\nabla \times \mathbf{J} = \mathbf{0}, \mathbf{J} \cdot \hat{\boldsymbol{n}} \neq 0$	**Magnetic type radiators** Supporting transverse currents $\nabla \times \mathbf{J} \neq \mathbf{0},\ \mathbf{J} \cdot \hat{\boldsymbol{n}} = 0$
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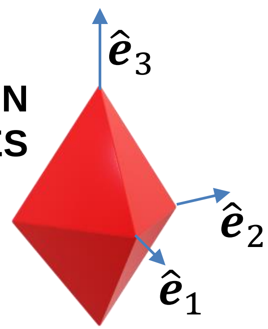
POLARIZABILITY MATRIX	$\boldsymbol{\gamma}_e = -\sum_{h=1}^{\infty} \chi_h^{\parallel} \mathbf{P}_{\mathrm{h}} \otimes \mathbf{P}_{\mathrm{h}}$	$\boldsymbol{\gamma}_m = \sum_{h=1}^{\infty} \kappa_h^{\perp} \mathbf{M}_{\mathrm{h}} \otimes \mathbf{M}_{\mathrm{h}}$
OPTIMAL CURRENT	$\boldsymbol{\gamma}_e \mathbf{p}_{\mathrm{opt}} = \gamma_{e,max} \mathbf{p}_{\mathrm{opt}}$ $\mathbf{j}_{\mathrm{opt}}(\mathbf{r}) = -\sum_{h=1}^{\infty} \chi_h^{\parallel} \left(\hat{\mathbf{p}}_{\mathrm{opt}} \cdot \mathbf{P}_h\right) \mathbf{j}_h^{\parallel}$	$\boldsymbol{\gamma}_m \mathbf{m}_{\mathrm{opt}} = \gamma_{m,max} \mathbf{m}_{\mathrm{opt}}$ $\mathbf{j}_{\mathrm{opt}}(\mathbf{r}) = \sum_{h=1}^{\infty} \kappa_h^{\perp} \left(\hat{\mathbf{m}}_{\mathrm{opt}} \cdot \mathbf{M}_h\right) \mathbf{j}_h^{\perp}(\mathbf{r})$
Qmin	$x^3 Q_{min} = \frac{6\pi}{\gamma_{e,max}}$	$x^3 Q_{min} = \frac{6\pi}{\gamma_{m,max}}$

Shape Dependent Q factor through Quasistatic MIMs: Symmetries

2 REFLECTION SYMMETRIES



Electric type radiators:
Supporting longitudinal currents
$\nabla \times \mathbf{J} = \mathbf{0}, \mathbf{J} \cdot \hat{\boldsymbol{n}} \neq 0$

Diple moments of the EQS modes aligned along $\hat{\boldsymbol{e}}_i$

$$\gamma_i = -\sum_{\substack{h \\ \mathbf{e}_i-\text{aligned}}} \chi_h^{\parallel} \left|\mathbf{P}_h\right|^2$$

$$x_h^3 Q_h^{\parallel} = \frac{6\pi}{(-\chi_h^{\parallel})|\mathbf{P}_h|^2}$$

Magnetic type radiators
Supporting transverse currents
$\nabla \times \mathbf{J} \neq \mathbf{0},\ \mathbf{J} \cdot \hat{\boldsymbol{n}} = 0$

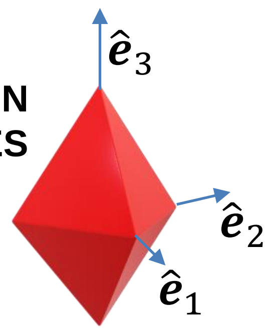
Diple moments of the MQS modes aligned along $\hat{\boldsymbol{e}}_i$

$$\gamma_i = \sum_{\substack{h \\ \mathbf{e}_i-\text{aligned}}} \kappa_h^{\perp} \left|\mathbf{M}_h\right|^2$$

$$x_h^3 Q_h^{\perp} = \frac{6\pi}{\kappa_h^{\perp}|\mathbf{M}_h|^2}$$

Shape Dependent Q factor through Quasistatic MIMs: Symmetries

2 REFLECTION SYMMETRIES



Electric type radiators:
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$$\gamma_i = - \sum_{h \atop \mathbf{e}_i-\text{aligned}} \chi_h^{\parallel} \, |\mathbf{P}_h|^2$$

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Magnetic type radiators
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$$\gamma_i = \sum_{h \atop \mathbf{e}_i-\text{aligned}} \kappa_h^{\perp} \, |\mathbf{M}_h|^2$$

$$x_h^3 Q_h^{\perp} = \frac{6\pi}{\kappa_h^{\perp}|\mathbf{M}_h|^2}$$

PARALLEL!

$$\frac{1}{(x^3 Q)_{\min}} = \sum_{h \atop \mathbf{e}_i-\text{aligned}} \frac{1}{x_h^3 Q_h^{\parallel}}$$

$$\frac{1}{(x^3 Q)_{\min}} = \sum_{h \atop \mathbf{e}_i-\text{aligned}} \frac{1}{x_h^3 Q_h^{\perp}}$$

UNIFORM MODES

$$\mathbf{j}_h^{\parallel} = \hat{\boldsymbol{e}}_i$$

$$\mathbf{j}_h^{\perp} = \hat{\boldsymbol{r}} \times \hat{\boldsymbol{e}}_i$$

Orthogonality: $k \neq h$

$$0 = \int_{\Omega} \mathbf{j}_h^{\parallel} \cdot \mathbf{j}_k^{\parallel} dV = \hat{\boldsymbol{e}}_i \cdot \mathbf{P}_k$$

$$\mathbf{P}_k = \mathbf{0}, \forall k \neq h \longrightarrow Q_{min} = Q_h$$

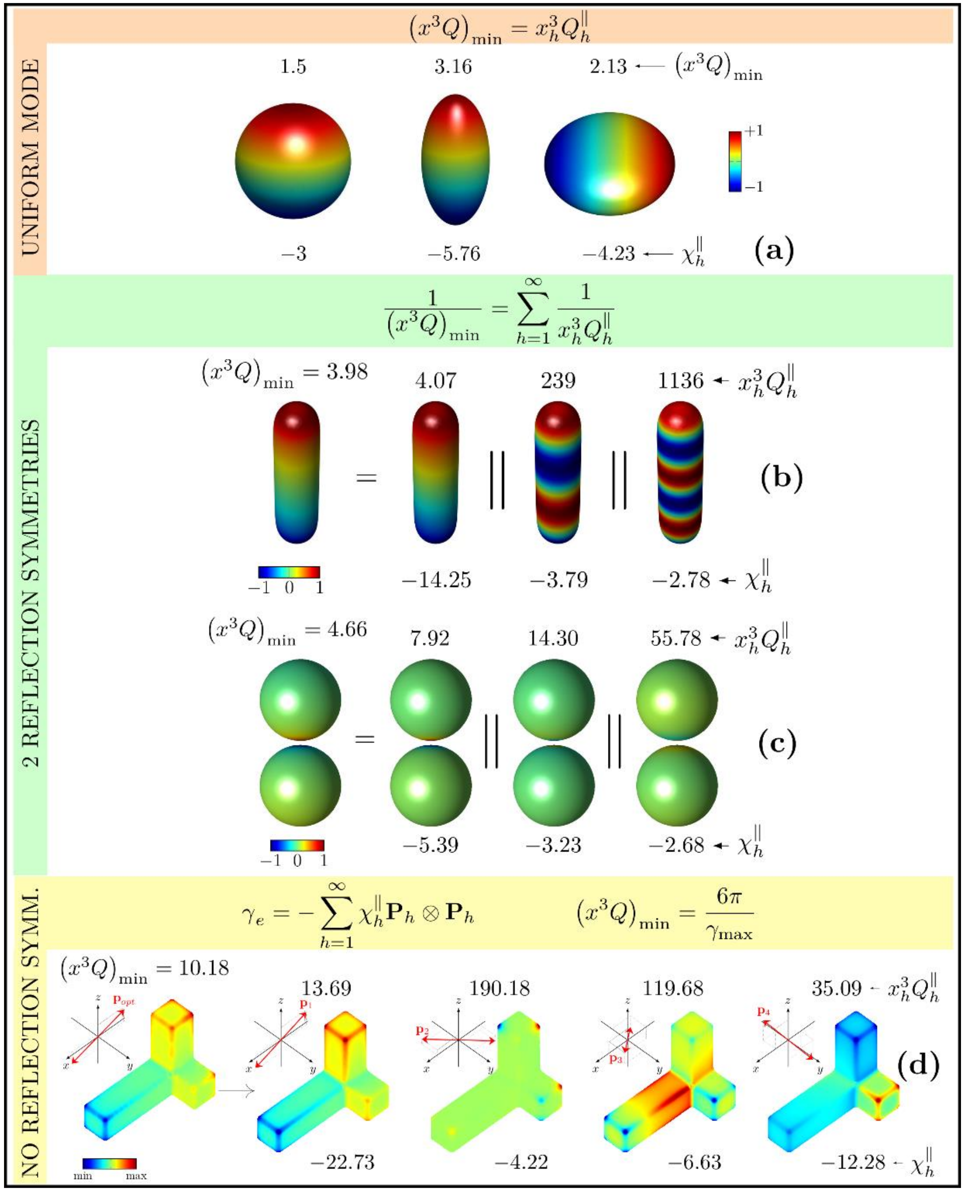
$$0 = \int_{\Omega} \mathbf{j}_h^{\perp} \cdot \mathbf{j}_k^{\perp} dV = \hat{\boldsymbol{e}}_i \cdot \mathbf{M}_k$$

$$\mathbf{M}_k = \mathbf{0}, \forall k \neq h \longrightarrow Q_{min} = Q_h$$

Ph.D itee
INFORMATION TECHNOLOGY ELECTRICAL ENGINEERING

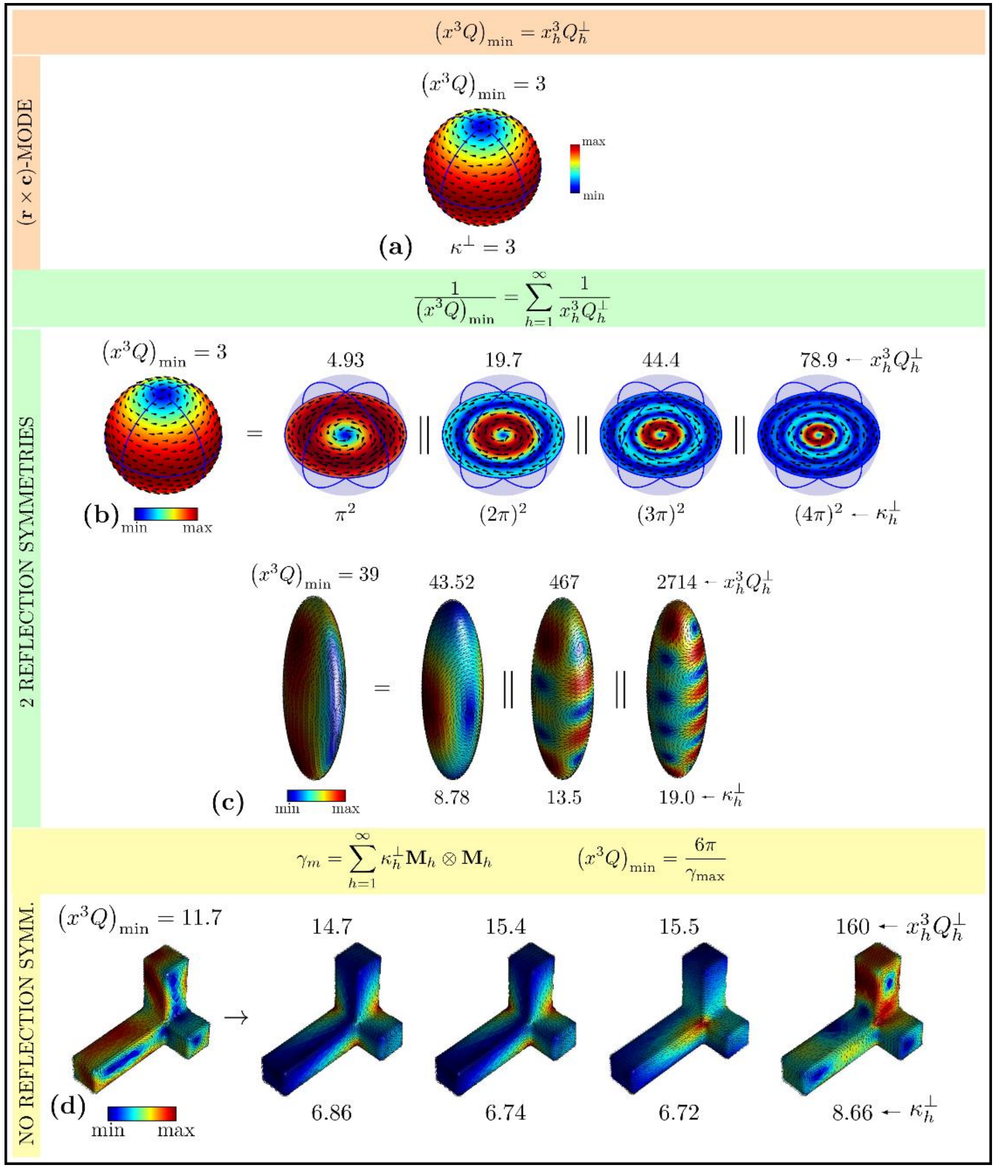
Mariano Pascale

Minimum Q: Electric Type Radiators



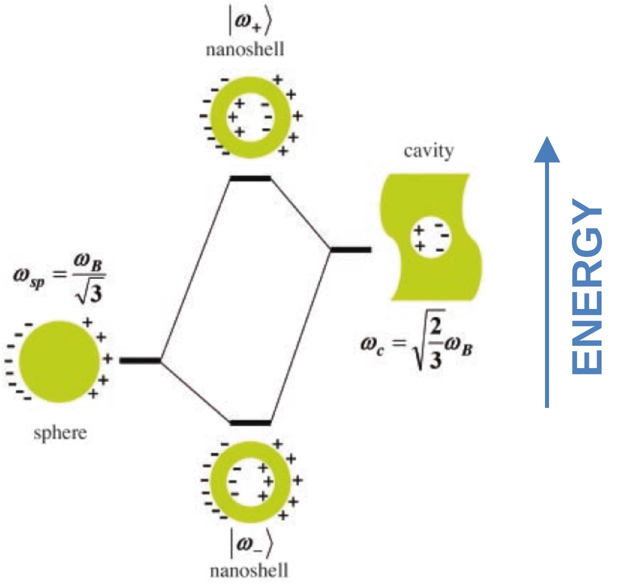
Mariano Pascale

Minimum Q: Magnetic Type Radiators



Complex Metal Nanostructures: Hybridization Model

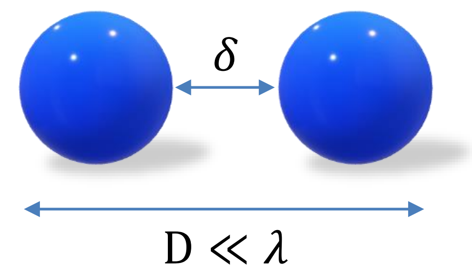
The plasmonic properties of complex metal-based nanostructures can be understood as the interactions of plasmons supported by metallic nanostructures of more elementary shapes.



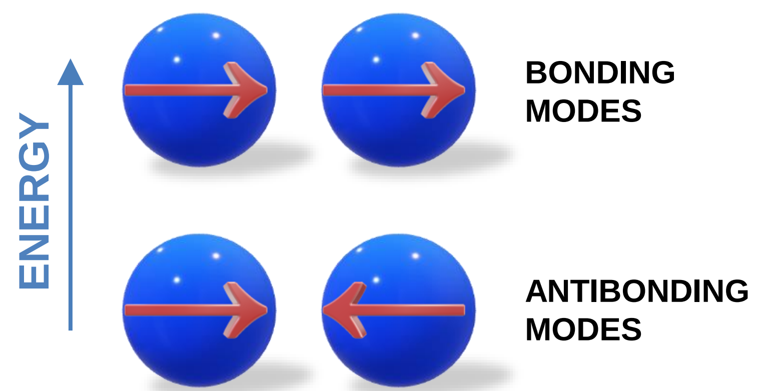
E. Prodan, C. Radloff, N. J. Halas, and P. Nordlander. "A hybridization model for the plasmon response of complex nanostructures." In: *Sci.* 302 (2003)

Nanoparticle Dimers: Plasmon Hybridization

- Dimer plasmons can be viewed as **bonding** and **antibonding** combinations of the individual nanoparticle plasmons.
- Plamon energies are evaluated as the interparticle separation decreases.
- This configuration corresponds to two interacting **dipole moments.**

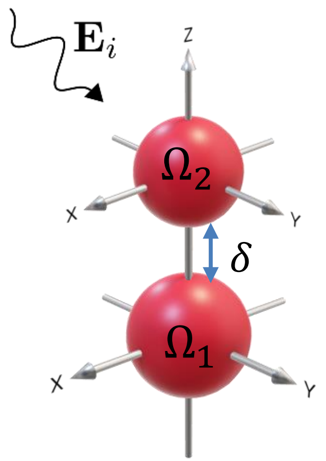


- The hybridization theory is limited to the electro-quasistatic regime
- Real-world metal structures have dimensions comparable to the incident wavelength.
- It cannot describe the radiative coupling between the spheres nor the existence of photonic modes (e.g. **magnetic modes**), which play a key role in the scattering from dielectric particles.



Plasmon Hybridization in Nanoparticle Dimers, P. Nordlander and, C. Oubre, E. Prodan, K. Li and, and M. I. Stockman, Nano Letters 2004 4 (5), 899-903

Full-Wave Hybridization through MIMs



Induced current in the sphere dimer:

$$\mathbf{J}(\mathbf{r}) = -\mathrm{i}\omega\varepsilon_0\chi \sum_{l=1}^{\infty} \frac{\gamma_l}{\gamma_l - \chi} \langle \mathbf{d}_l, \mathbf{E}_{inc} \rangle \mathbf{d}_l^{\mathrm{DIMER}}(\boldsymbol{r})$$

$$\mathbf{d}_{\mathrm{l}}^{\mathrm{DIMER}}(\boldsymbol{r}_{\mathrm{j}}) = \sum_{\boldsymbol{q}} h_l^q \ \mathbf{d}_{\mathrm{q}S}^{\mathrm{SPHERE}}(\boldsymbol{r}_{\mathrm{j}}), \qquad \boldsymbol{r}_j \in \Omega_j, j = 1{,}2$$

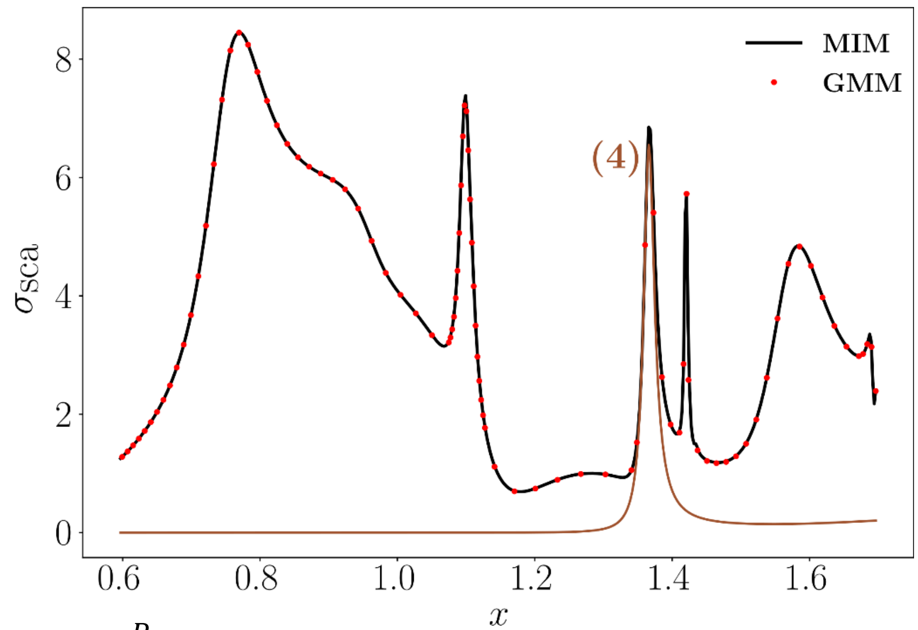
HYBRIDIZATION WEIGTHS:

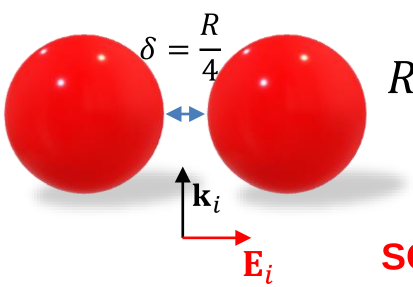
- Measure the contribution of the single sphere modes
- Depend only on the radii of the spheres and the gap size: $h_l^q = h_l^q(R_1, R_2, \delta)$

Full-wave electromagnetic modes and hybridization in nanoparticle dimers. Pascale, M., Miano, G., Tricarico, R., Forestiere, C. Sci Rep 9, 14524 (2019)

Silicon Dimer

$\varepsilon_R = 16$

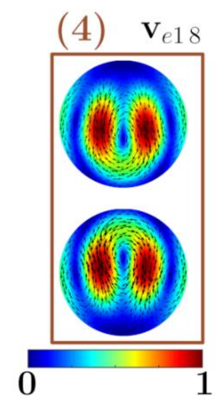




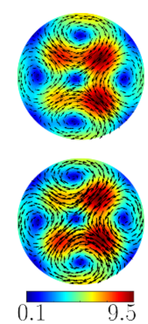
$$R_1 = R_2 = R$$

$$x = \frac{2\pi R}{\lambda}$$

SCATTERED ELECTRIC FIELD



DOMINANT MODES



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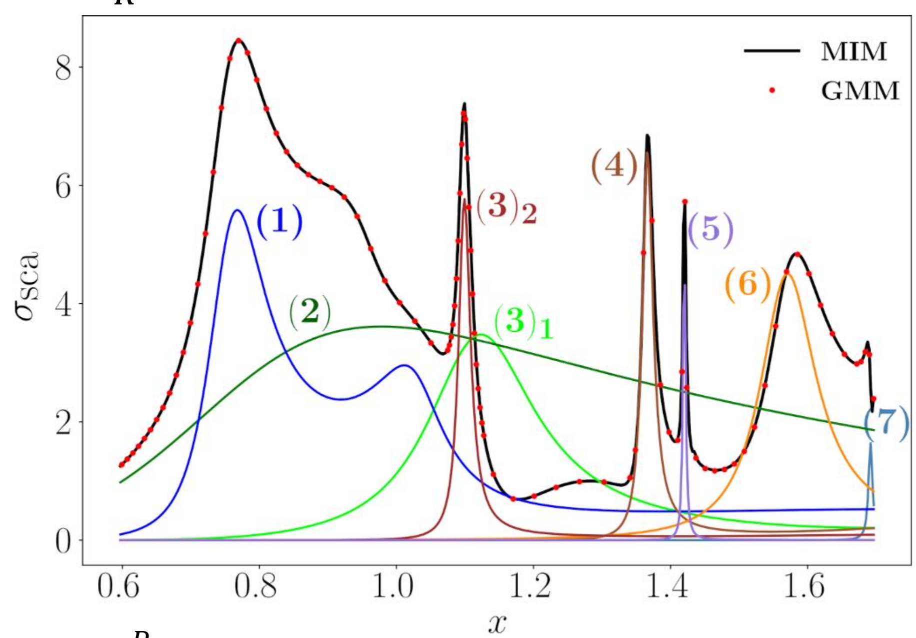
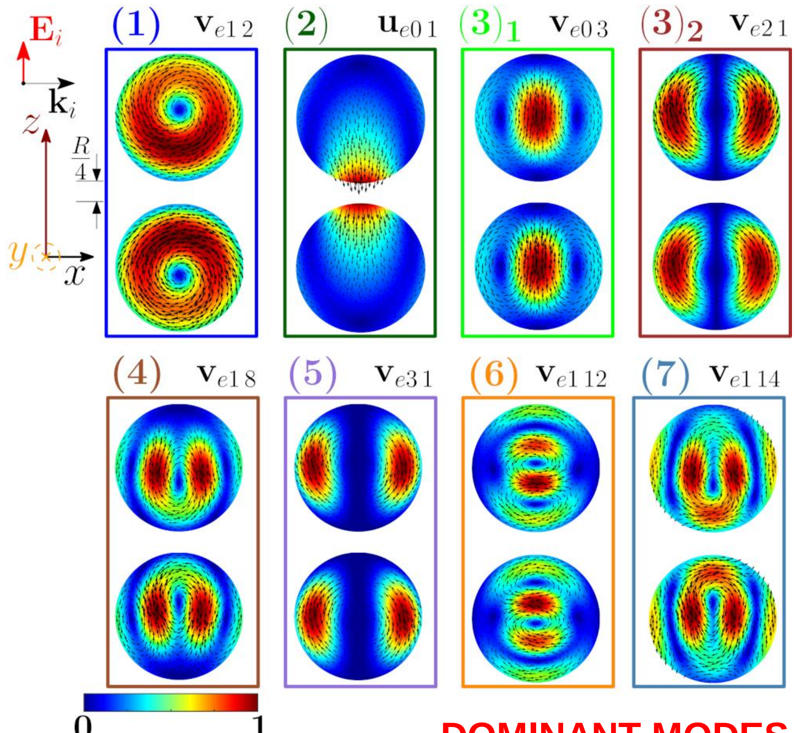
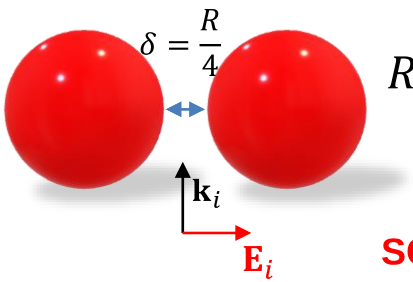




DOMINANT MODES



$$R_1 = R_2 = R$$

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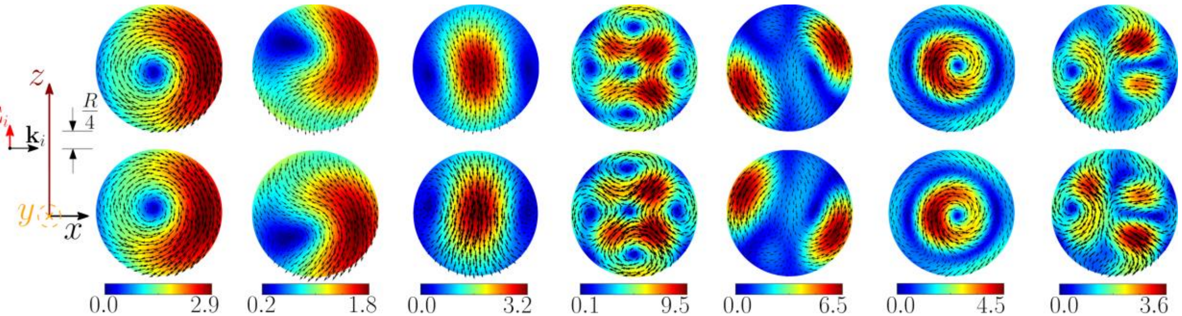
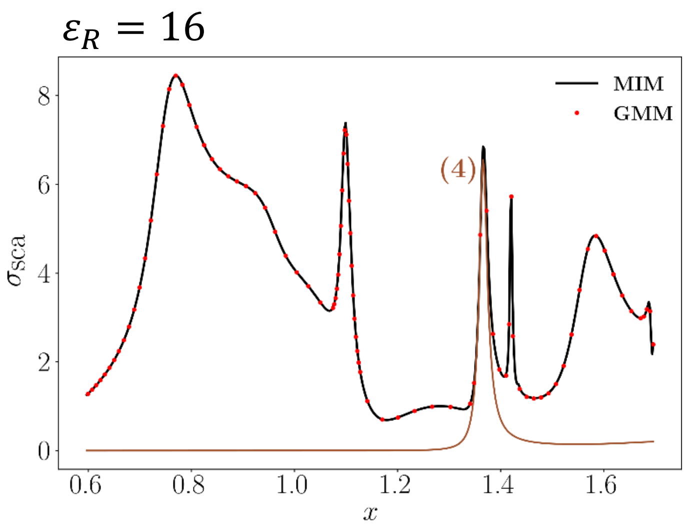
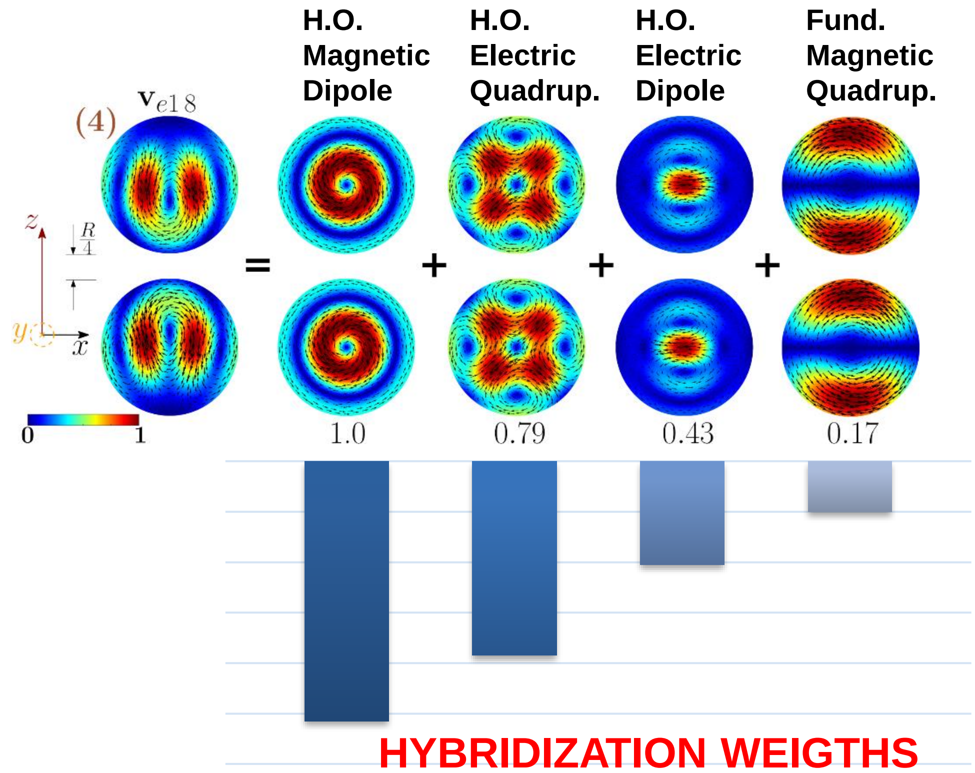
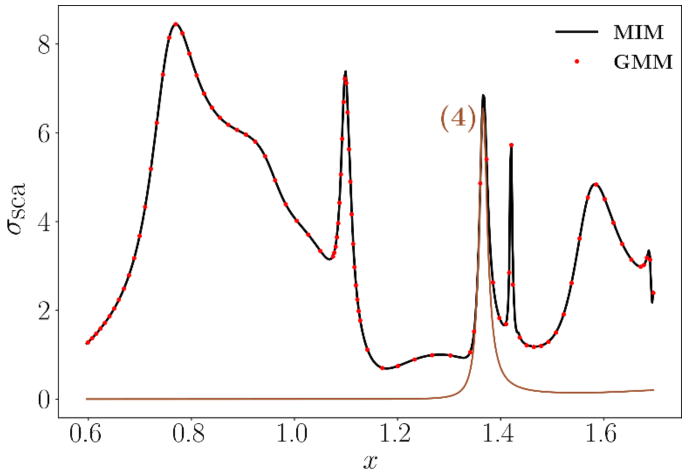


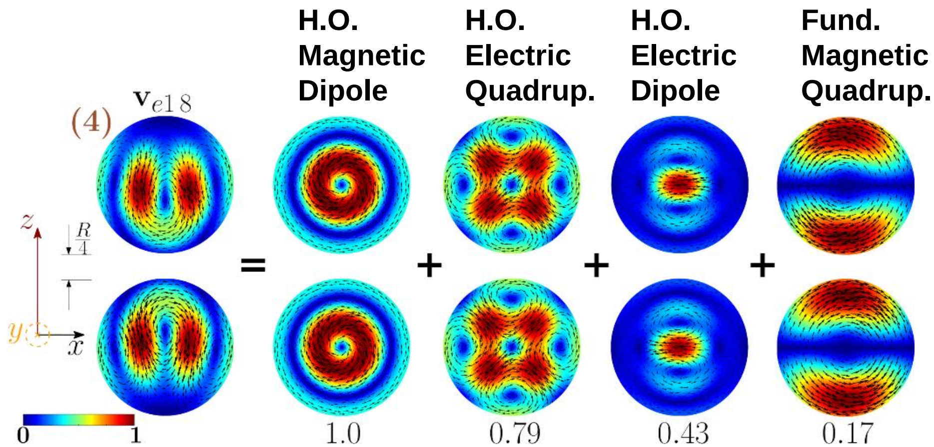
Modes Hybridization

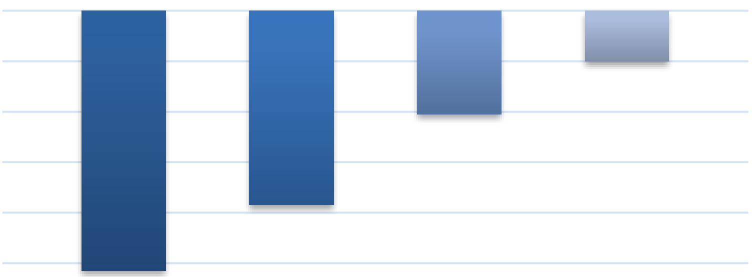




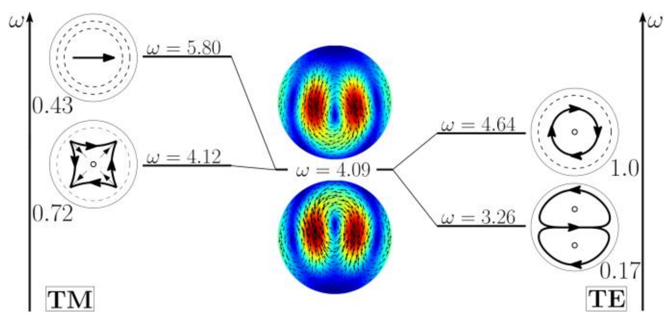
Modes Hybridization







HYBRIDIZATION WEIGTHS



FULL-WAVE HYBRIDIZATION DIAGRAM

Conclusions

- A spectral theory for the description of the resonances in metal and dielectric nanostructures, when they are much smaller than the wavelength, has been introduced: **quasistatic MIMs.**
- The quasistatic description has been extended to include the radiation for electrically small objects, through a perturbative approach: **modes Q** factor.
- This framework has been exploited for a **new** strategy aimed at the calculation of the **optimal currents** yelding the **minimum Q** factor supported by an arbitrary shaped radiator.
- A **full-wave** spectral theory for the description of resonances in objects comparable with the operating wavelength has been introduced.
- A **new full-wave hybridization theory** for the electromagnetic scattering from metal and dielectric sphere dimers has been presented.