Elena Napoletano

Tutor: Franco Garofalo
XXX Cycle - III year presentation

Informational Cascade as a Pinning Control Problem

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SINCRO Nonlinear Systems, Networks and Control

Motivation

"In the face of the crisis, we felt abandoned by conventional tools... we need to develop ***complementary tools*** *to improve robustness of our overall framework... I would very much welcome inspiration from other disciplines:* ***physics, engineering, biology****. Bringing experts form these fields together with economist and central bankers is potentially very creative and valuable. Scientists have developed sophisticated tools for analysing* ***complex dynamic systems*** *in rigorous way."*

Jean-Claude Trichet

former European Central Bank Governor

ECBs flagship annual Central Banking Conference, 2010

Informational Cascades

An informational cascade is an imitation phenomenon which can emerge in financial markets.

- An individual, having observed the actions of his peers, can decide to follow their behavior without regard to his own information.
- This behavior can trigger the informational cascade in which each agent blindly replicates the trading strategy of the other traders.

Existing Models of Informational Cascades

They are static models: they do not capture the **dynamic** behavior and the **learning** capabilities of the agents.

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The behavior of just the first few individuals generates almost surely an informational cascades which involves **all** the subsequent agents.
This result disregards empirical evidence showing that information may spread among the agents with different **intensities** [2].

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This result disregards empirical evidence showing that information may spread among the agents with different **intensities** [2].

The agents get access to trading one by one. The sequence of the trade is exogenously given, thus the **influence** among the agents are completely **arbitrary**.

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Scope

Our aim is to propose a **dynamic model** which captures the relationship skills of the agents: each agent develops a certain **opinion** interacting with his peers and **learning** from their actions.

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This model should be able of replicating *partial* informational cascades, that is, cascades of different **intensities** which do not involve all the agents, in line with empirical evidence.

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This model should be able of replicating *partial* informational cascades, that is, cascades of different **intensities** which do not involve all the agents, in line with empirical evidence.

The influences among the agents should not be randomic and sequential, but should account for a specific **pattern of relationships**.

A new Model of Opinion Dynamics [3]

Agent i dynamically develops a certain opinion $x_i(k)$.

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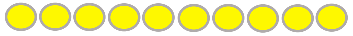
Exogenous sequence



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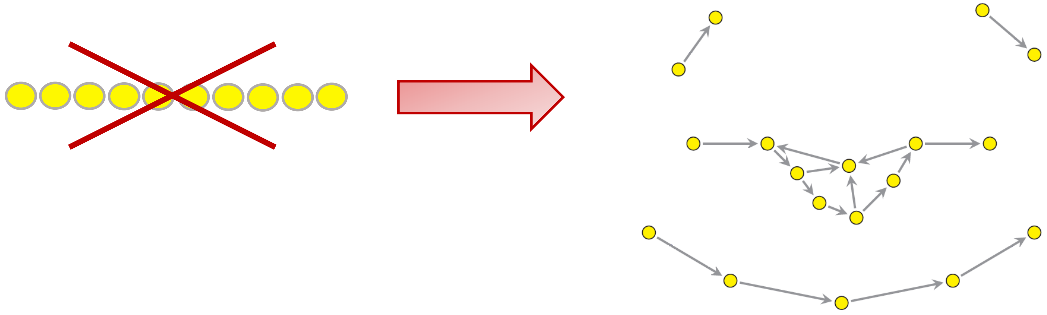

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Diffusive coupling

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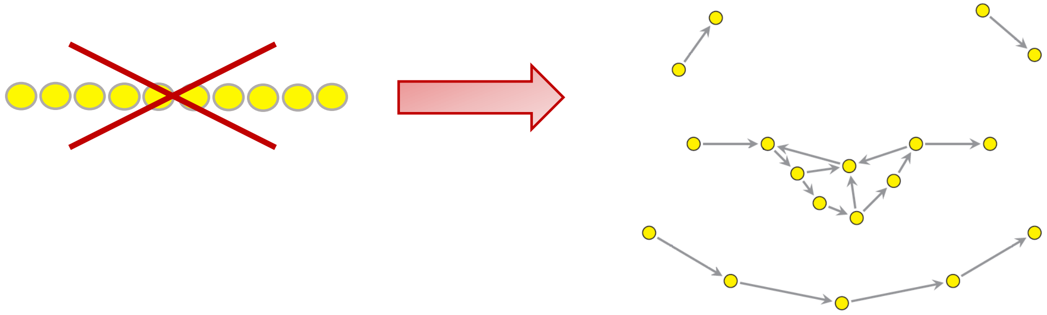

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Set of neighbors of agent i

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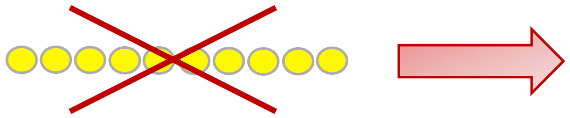

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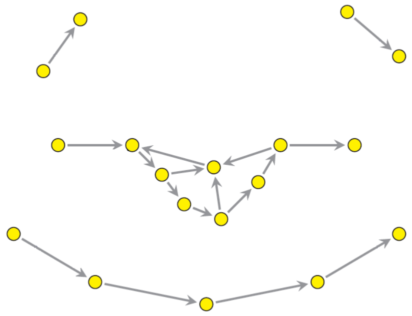

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Generic element of the adjacency matrix

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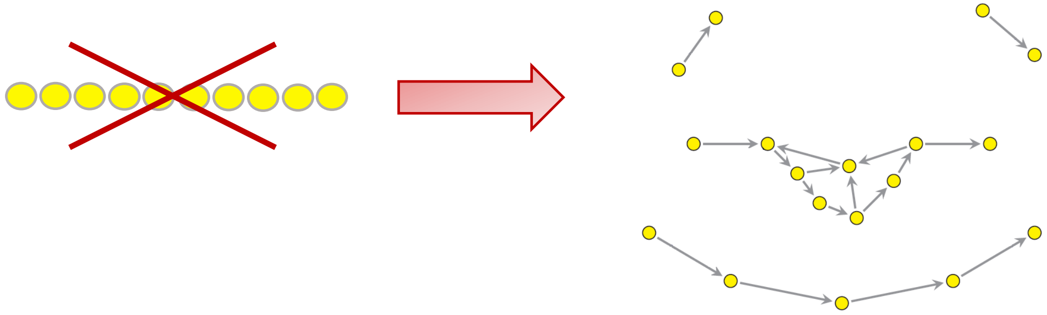

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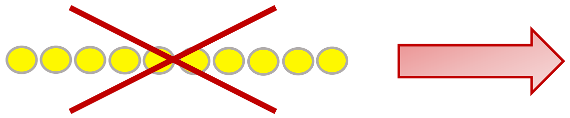

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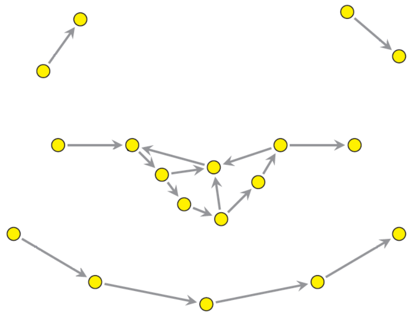

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Agent's actions

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Exogenous factors

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A new Model of Opinion Dynamics [3]

The resulting model is

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We can think at the trading strategy $s_i(k)$ of agent i as an output of his opinion:

$$s_i(k) = g(x_i(k))$$

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We can see the information $\bar{x}$ as a **virtual agent** who exerts a **control action** on the subset I of agents, which become **leaders/informed** and thus perform the «correct» trading strategy:

$$\bar{s} = g(\bar{x})$$

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In the view of this, we can notice the similarity with **Pinning Control**, in which the virtual agent is the «pinner», and the informed agent are the so called «pinned nodes».

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Pinning Control [4]

It is a control strategy which allows to drive a network of coupled dynamical systems from any initial state to a desired synchronous state $\bar{\boldsymbol{x}}$, i.e., that of the *pinner*, by applying local control actions to a small subset P of nodes, called *pinned nodes*.

The generic equation of a pinning controlled dynamical network is

$$x_i(k+1) = f\big(x_i(k)\big) + c \sum_{j \in N_i} a_{ij} \Big(x_j(k) - x_i(k)\Big) + \kappa\delta_i\big(\bar{x} - x_i(k)\big)$$

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Intrinsic dynamics of node i

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Pinner's control action

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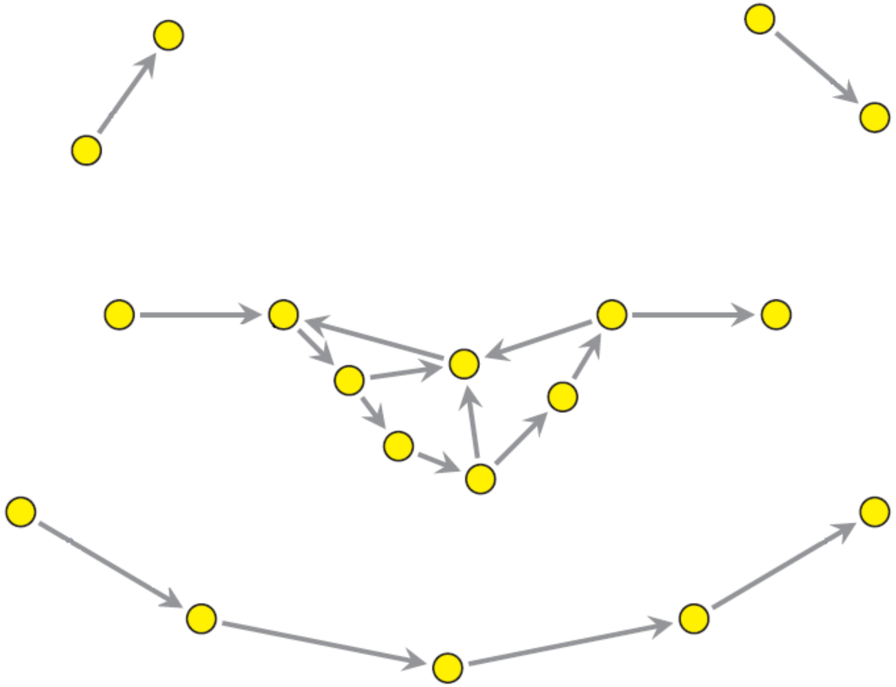

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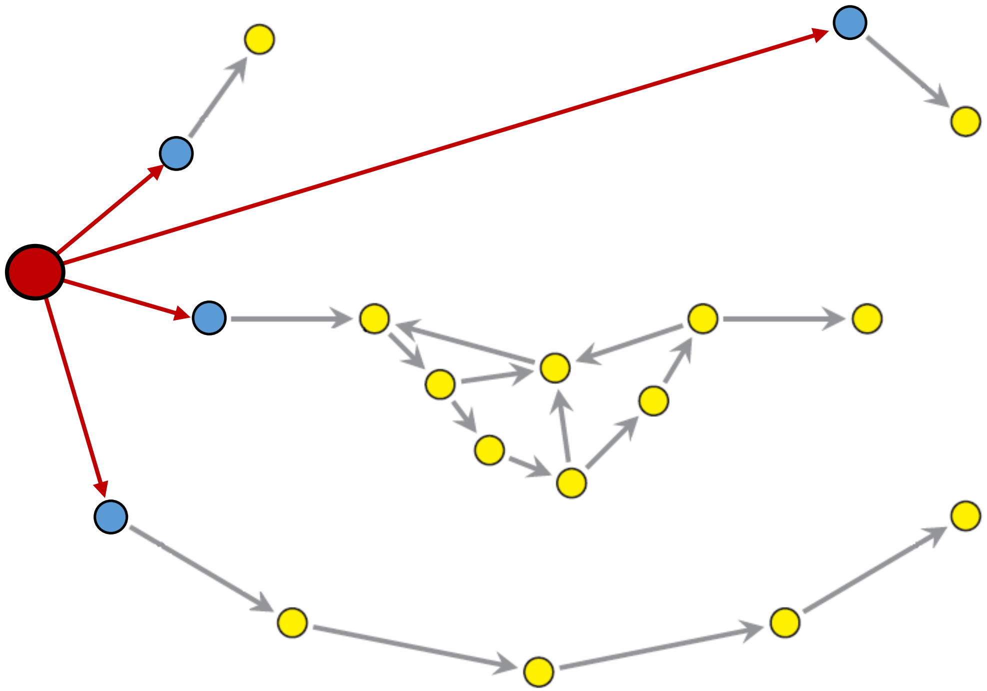

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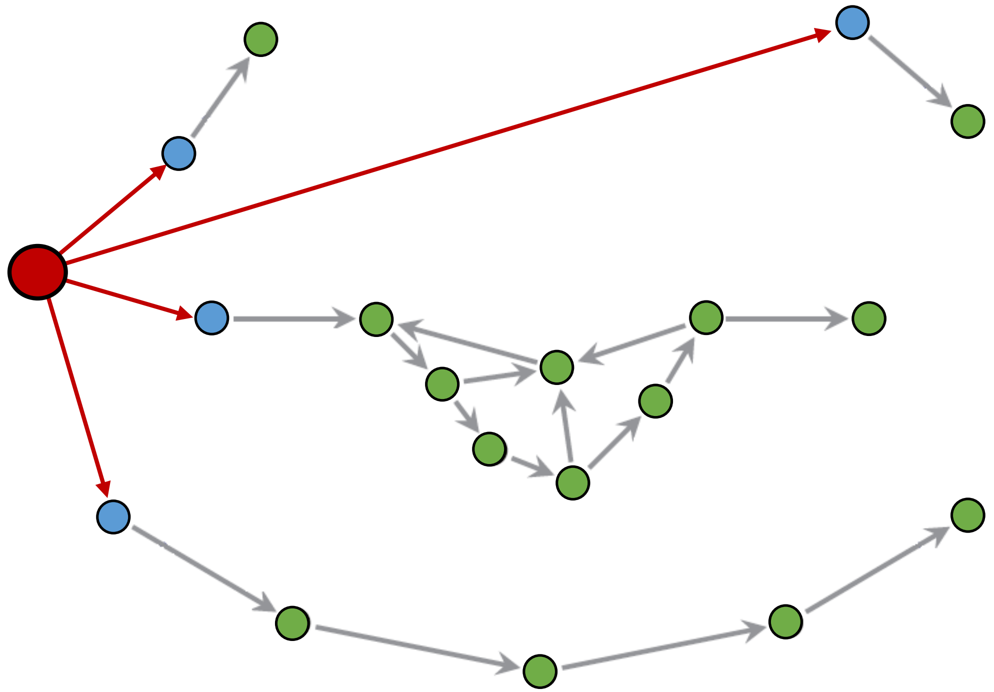

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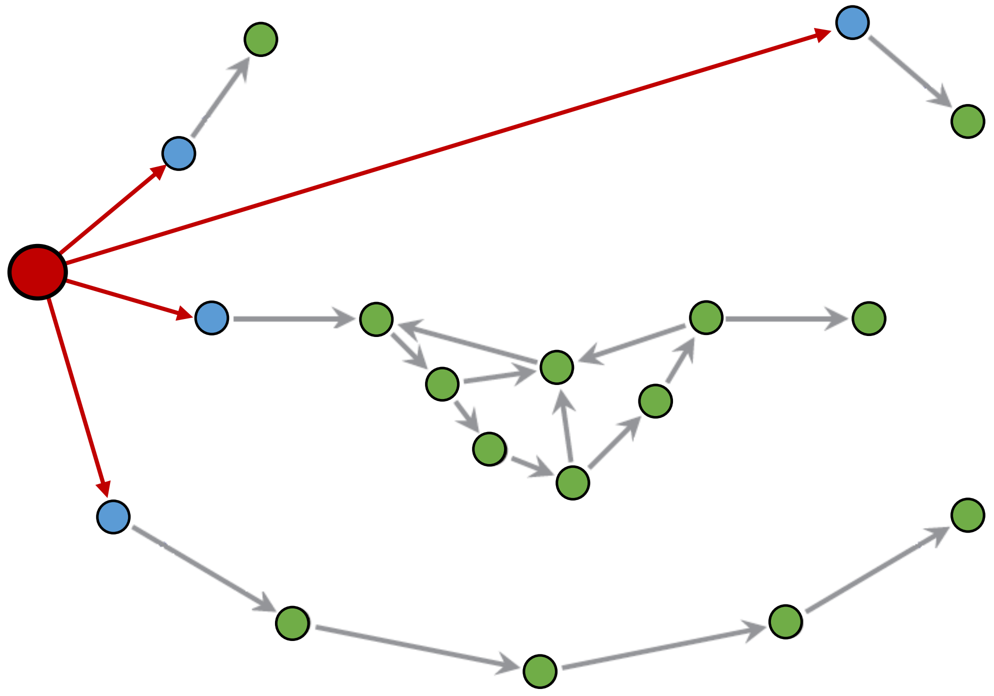
TOTAL INFORMATIONAL CASCADE



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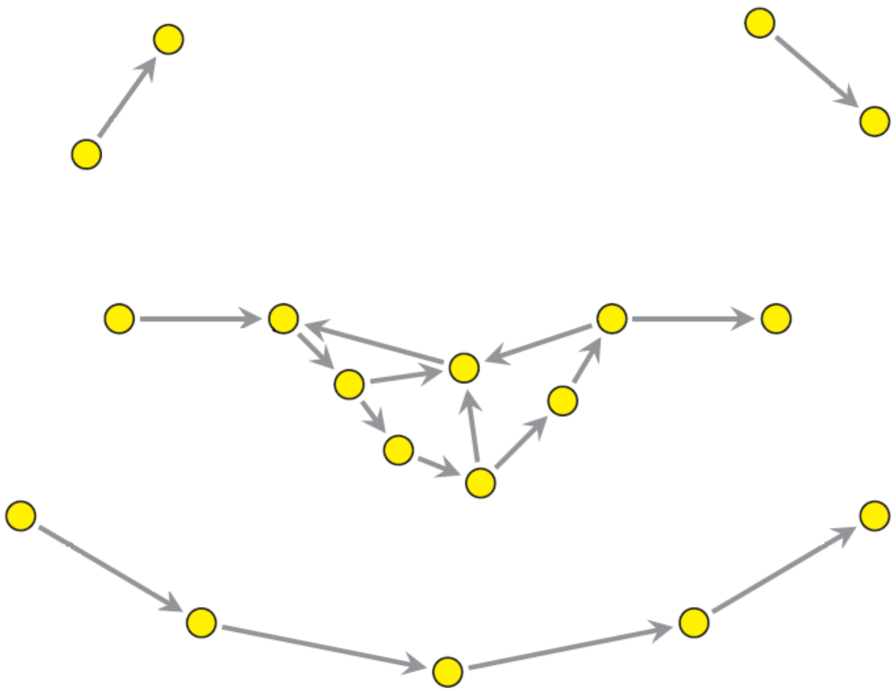

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[5] The partial pinning control strategy for large complex networks, Pietro De Lellis, Franco Garofalo, Francesco Lo Iudice, Automatica (to appear)

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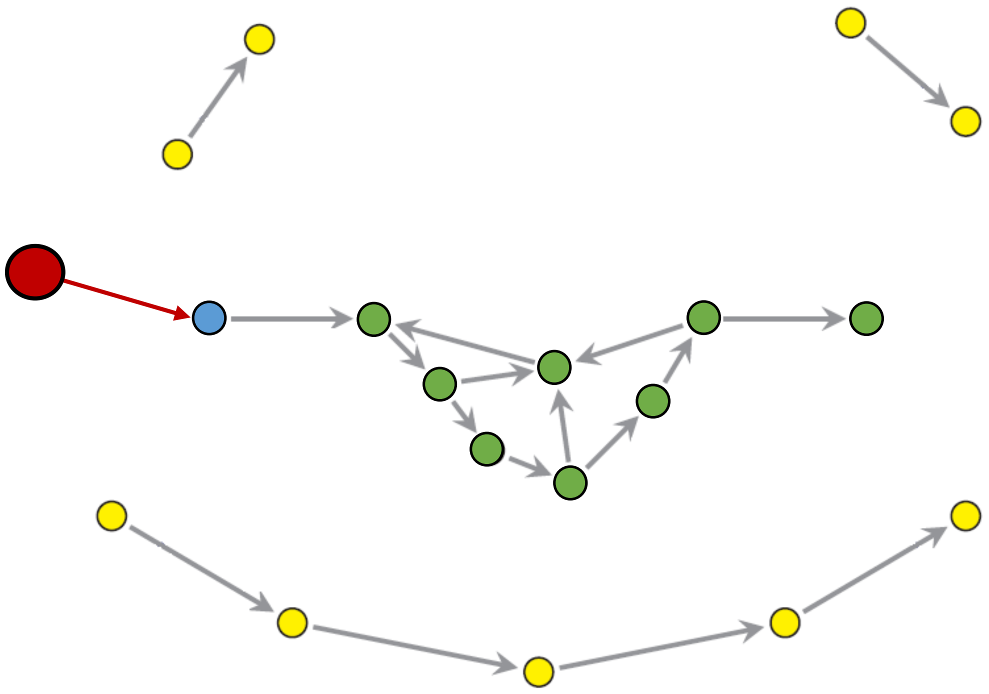

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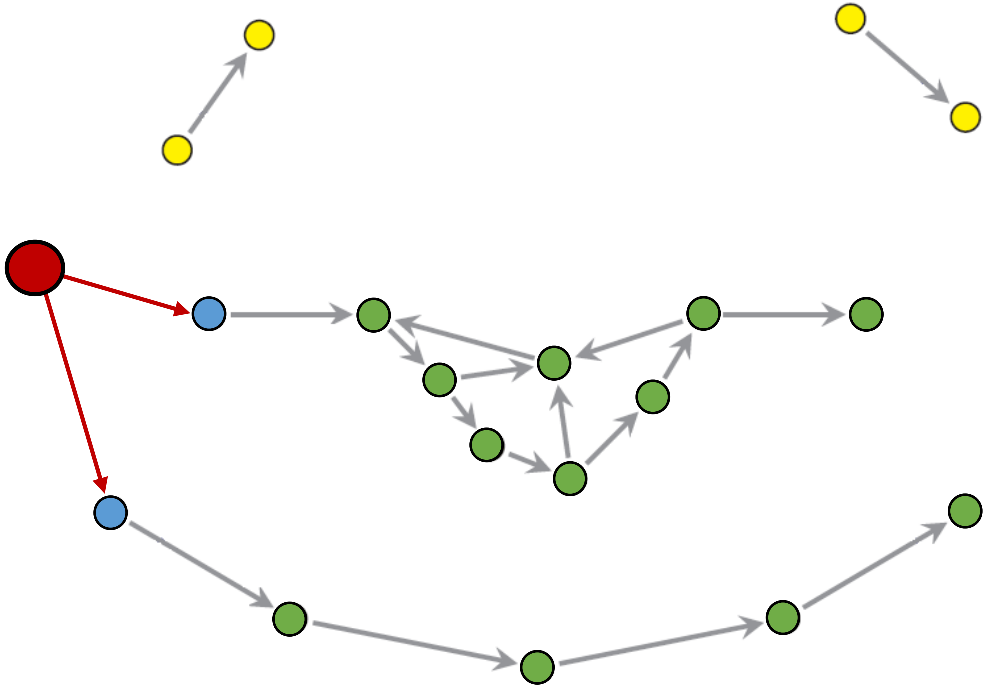

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The network

$$x_i(k+1) = f\big(x_i(k)\big) + c \sum_{j \in N_i} a_{ij}\Big(x_j(k) - x_i(k)\Big) + \kappa\delta_i\big(\bar{x} - x_i(k)\big)$$

is said to be *q-partially* pinning controlled to the pinner's trajectory when

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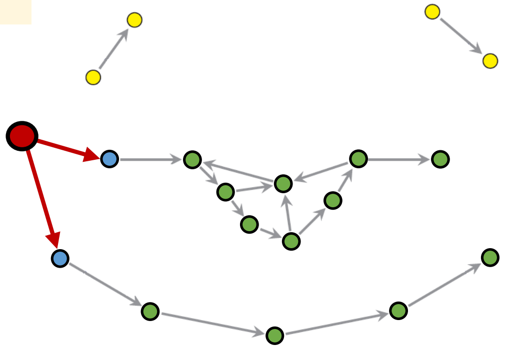
PARTIAL INFORMATIONAL CASCADE

$Q =$ set of controllable nodes



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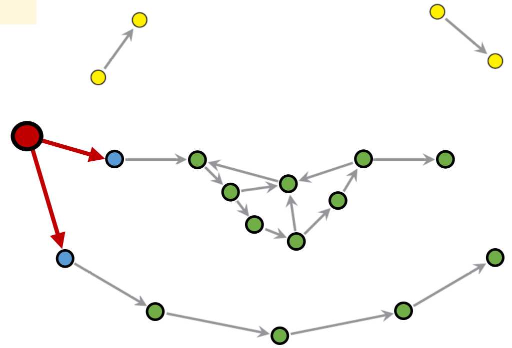
Under appropriate assumptions on the nodes' dynamics, **topological conditions** which ensure the partial pinning controllability of the system are provided.



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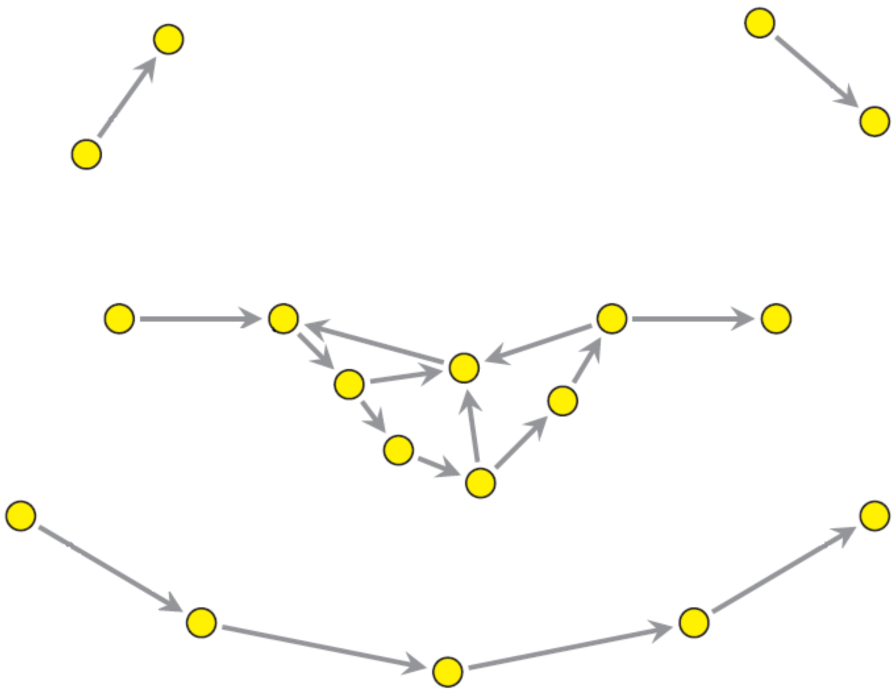
Some Useful Graph Theoretical Tools

- A complex network can be seen from a macroscopical point of view as an ensemble of **Connected Components**.

- A Strongly Connected Component is a subset of nodes in which there exists a directed path between any two chosen vertices.

- A root is a node with only outgoing edges.



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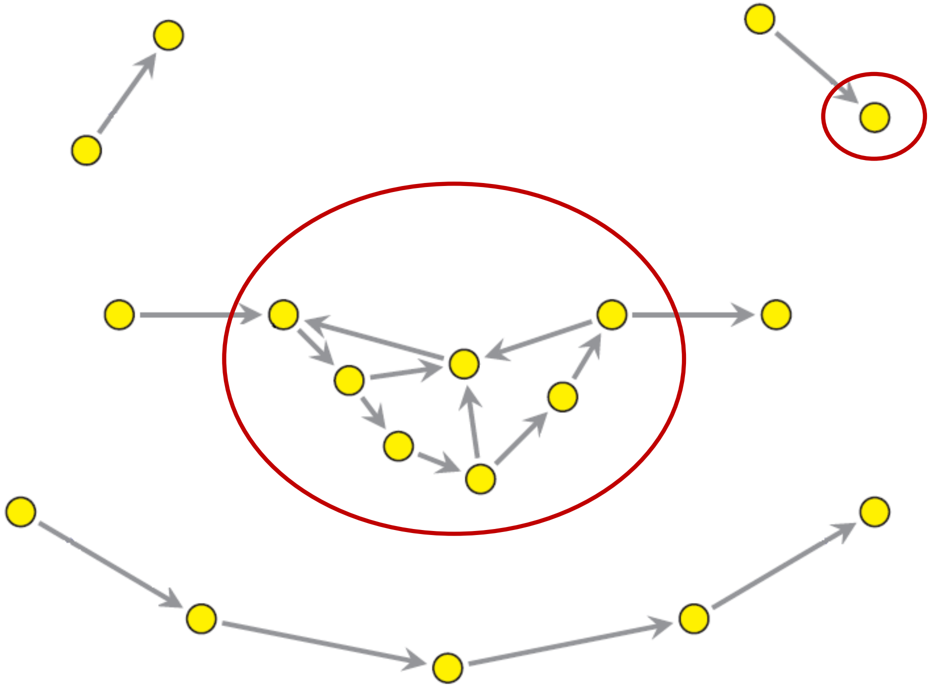

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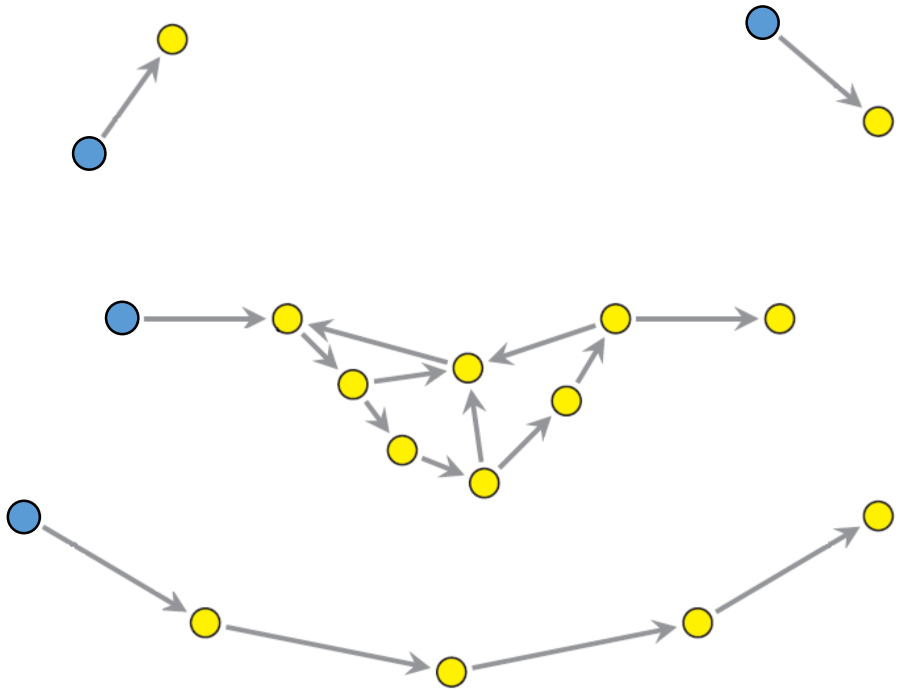
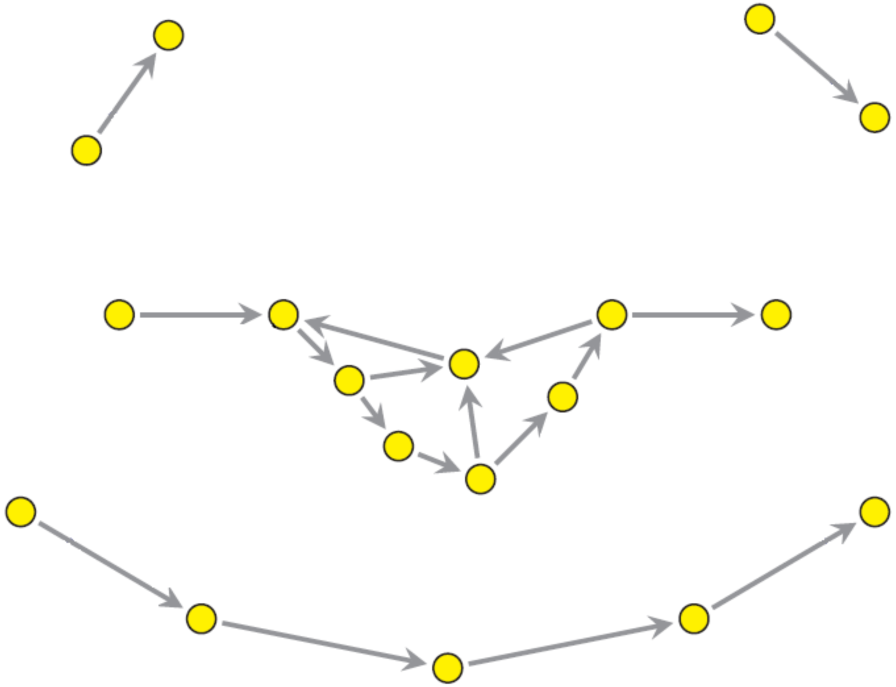


Topological Conditions for Partial Pinning Controllability [5]

A SCC is pinning controllable if all the roots in its upstream are pinning controlled.



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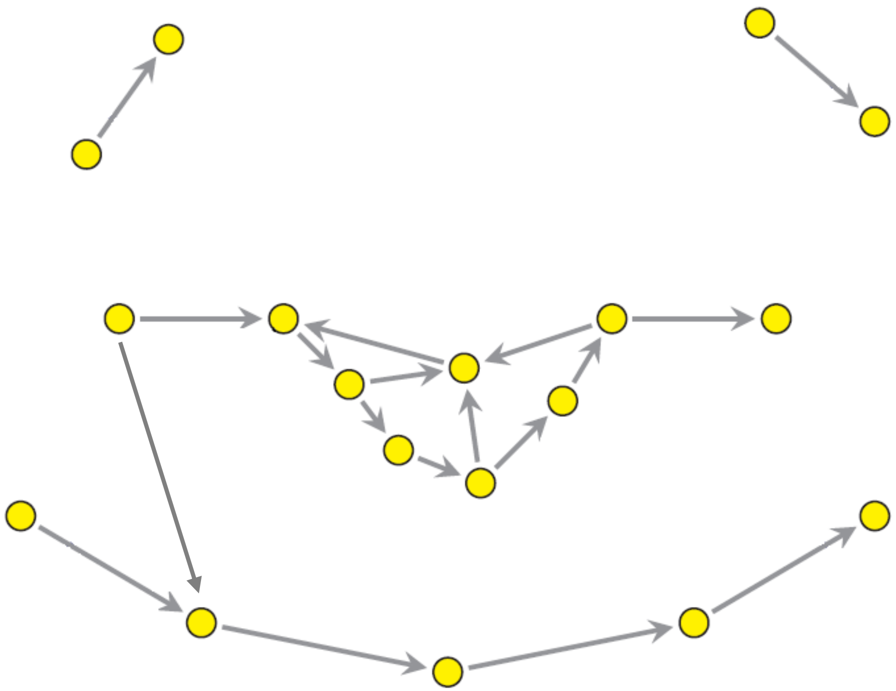

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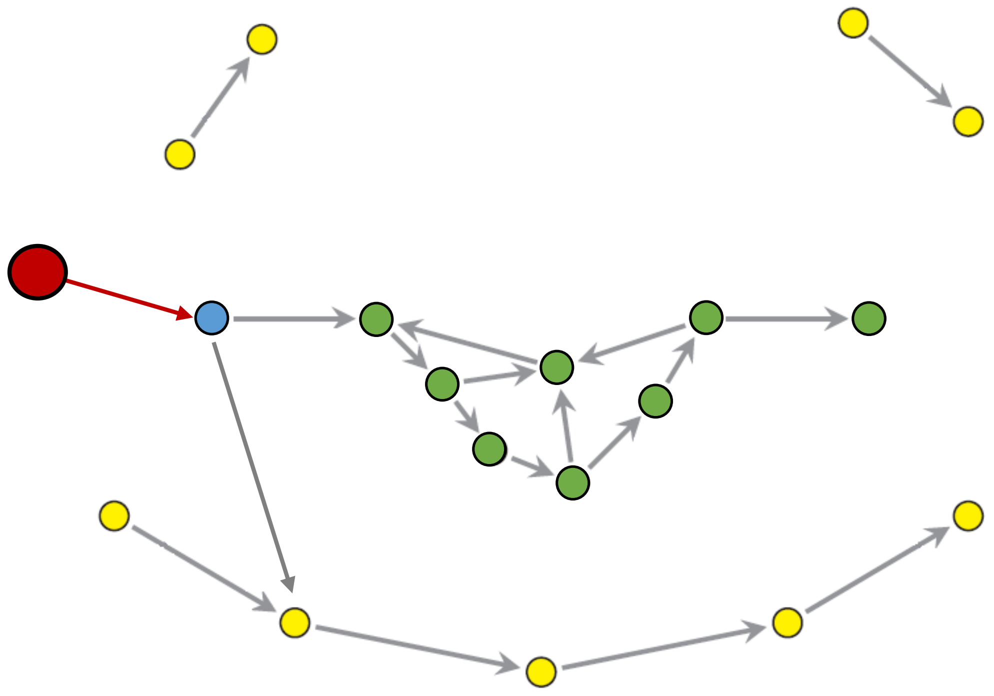

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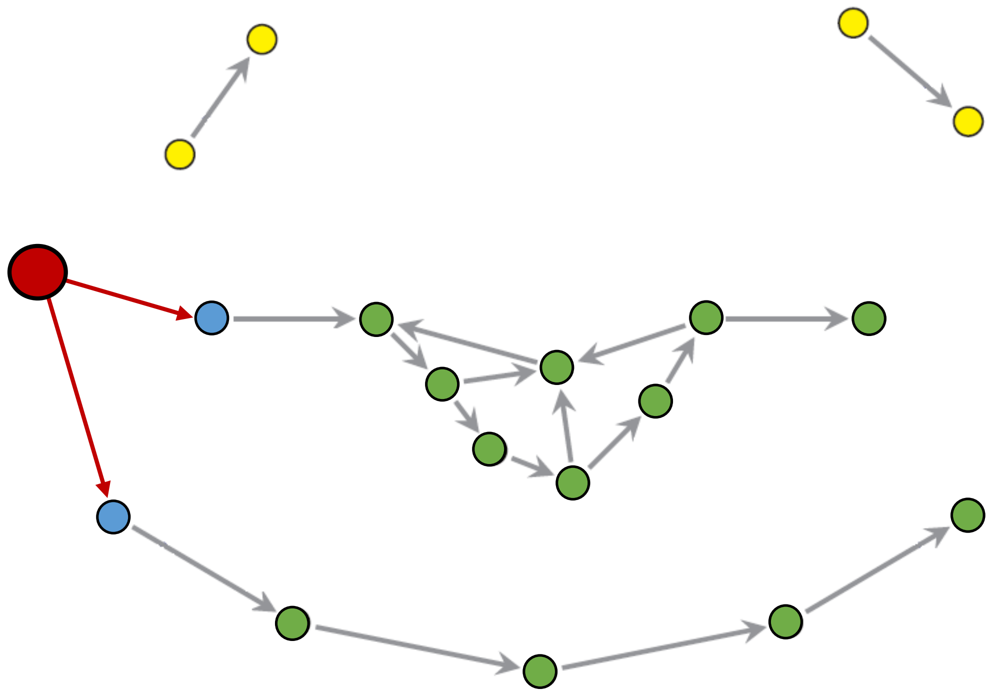
Selection of Pinned Nodes

The **partial pinning control algorithm** [5] tells us which nodes should be pinned in order to maximize the number of controllable nodes $|Q|$, that is, the nodes whose trajectory converges to that of the pinner, given the number p of pinned nodes.

$$q^* = \max_P |Q|$$
$$|P| = p$$



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Differences and Similarities

$$\begin{cases} x_i(k+1) = x_i(k) + \sum_{j\in N_i} \alpha_{ij}\big(r(k)\big)\Big(x_j(k) - x_i(k)\Big) + \delta_i\big(\bar{x} - x_i(k)\big) \\ r_i(k+1) = f\Big(r_i(k), \beta^{l^*}, \tau(k)\Big) \end{cases}$$

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- Our model of opinion dynamics presents some differences compared to that of pinning control.
- However, we can rely on the **topological conditions** for partial pinning controllability to predict the nodes which *should* reach consensus on the pinner’s opinion, and thus are involved in the informational cascade.

Opinion Dynamics and Informational Cascades

- We assumed that the opinion of the agent reflects in his trading strategy:

$$s_i(k) = g(x_i(k))$$

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We test the capability of our model of triggering partial informational cascades in an agent-based model of artificial financial market.

The Agent-Based Financial Market [6]

Heterogeneous Agents

$x_i(k)$: current **opinion** on the expected return of each asset.
$r_i(k)$: agent's **reputation**. It coincides with his current wealth.

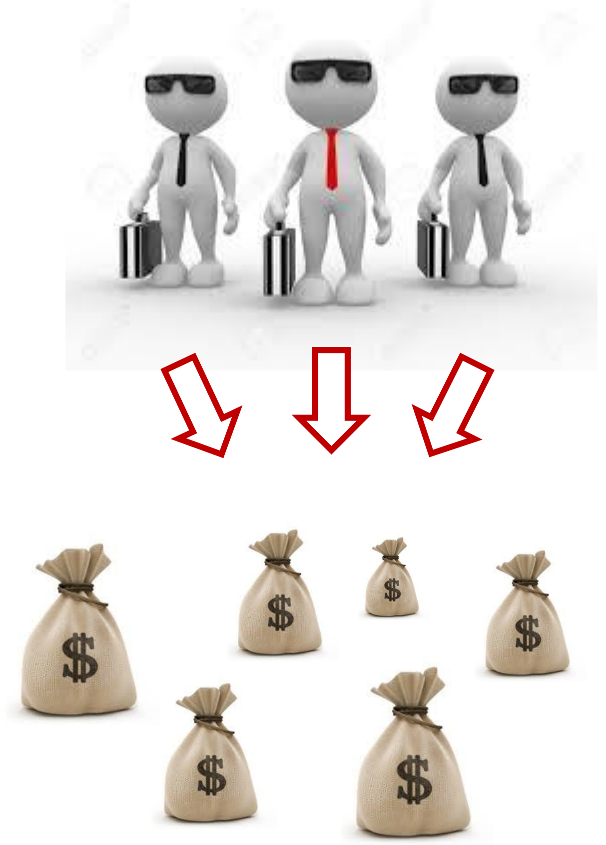
Set of Financial Assets

Each asset is characterized by a limited availability and fixed expected returns $\bar{\boldsymbol{x}}$, which coincides with the **information** available to the informed agents.

[6] DeLellis, P., Garofalo, F., Iudice, F. L., & Napoletano, E. (2015). Wealth distribution across communities of adaptive financial agents. *New Journal of Physics*, *17*(8), 083003.

The Trading

The strategy $s_i(k) = g(x_i(k))$ defines the agent's preference about the assets.



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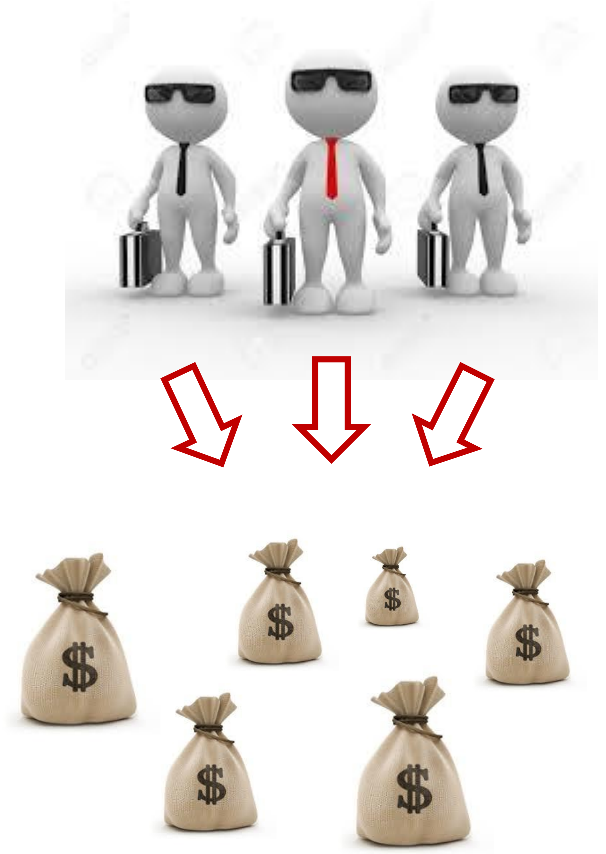

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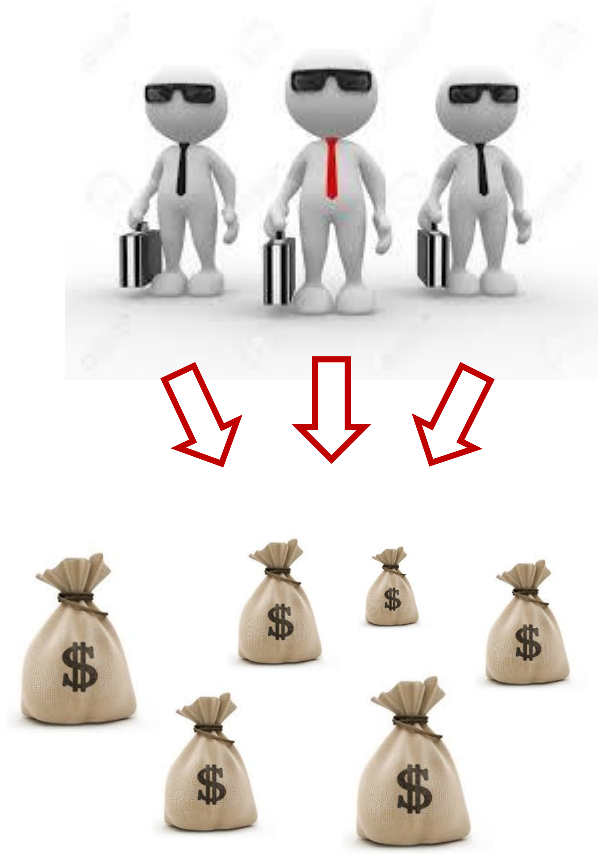
The Trading

The strategy $s_i(k) = g(x_i(k))$ defines the agent's preference about the assets.

The trading is a stochastic process.

The current wealth is updated depending on the outcome of the trading and on the application of a tax.



Numerical Setup

We perform p simulations, where p varies between 0 and the minimum number of nodes which are required to be pinned in order to control the whole network.

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We perform p simulations, where p varies between 0 and the minimum number of nodes which are required to be pinned in order to control the whole network.

We select the p pinned nodes according to the partial pinning control algorithm. The fraction of controllable nodes $Q(p)$ returned by the algorithm *could* be a **prediction** of the triggered informational cascade.

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Measured Parameters

- For each simulation p, we measure
  - The **intensity** of the **triggered informational cascade** as

$$H(p) = \frac{|\{i \in V : s_i(k) = \bar{s}, k > k^*\}|}{\mathrm{N}}$$

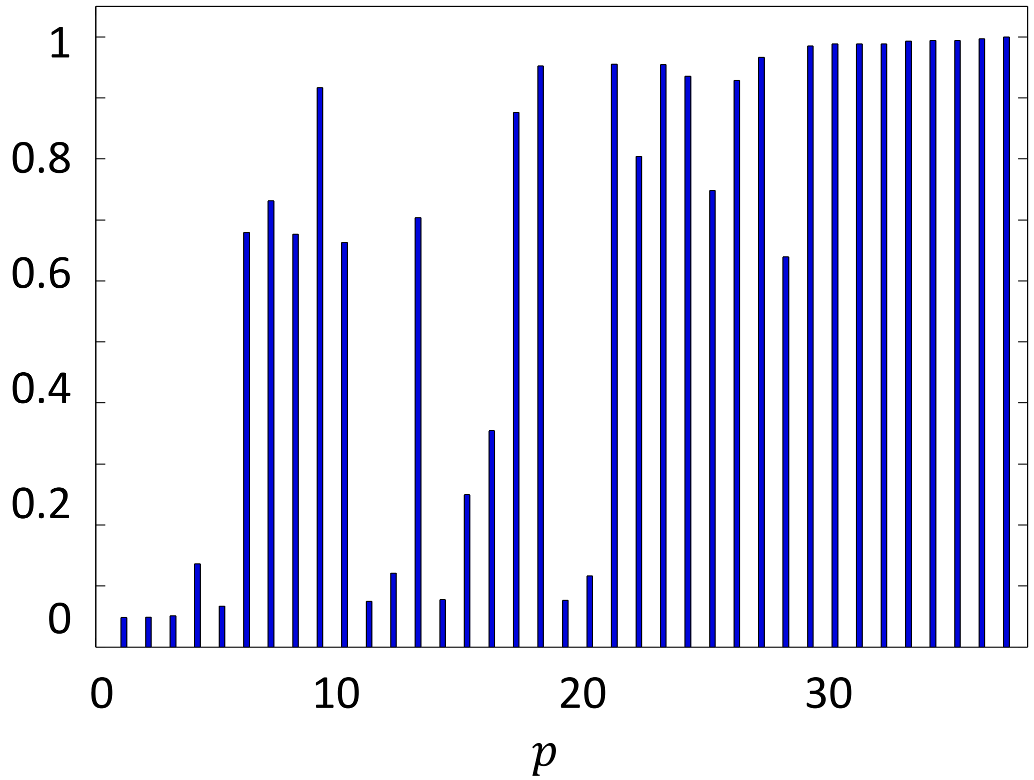
  - The fraction of **controlled nodes** as the fraction of agents who reach consensus both on the opinion and on the trading strategy of the pinner:

$$Q^o(p) = \frac{|\{i \in V : x_i(k) = \bar{x}, k > k^*\}|}{\mathrm{N}}$$

- We compare these measurements with the fraction of **controllable nodes** $Q(p)$ returned by the partial pinning control algorithm.

Main results

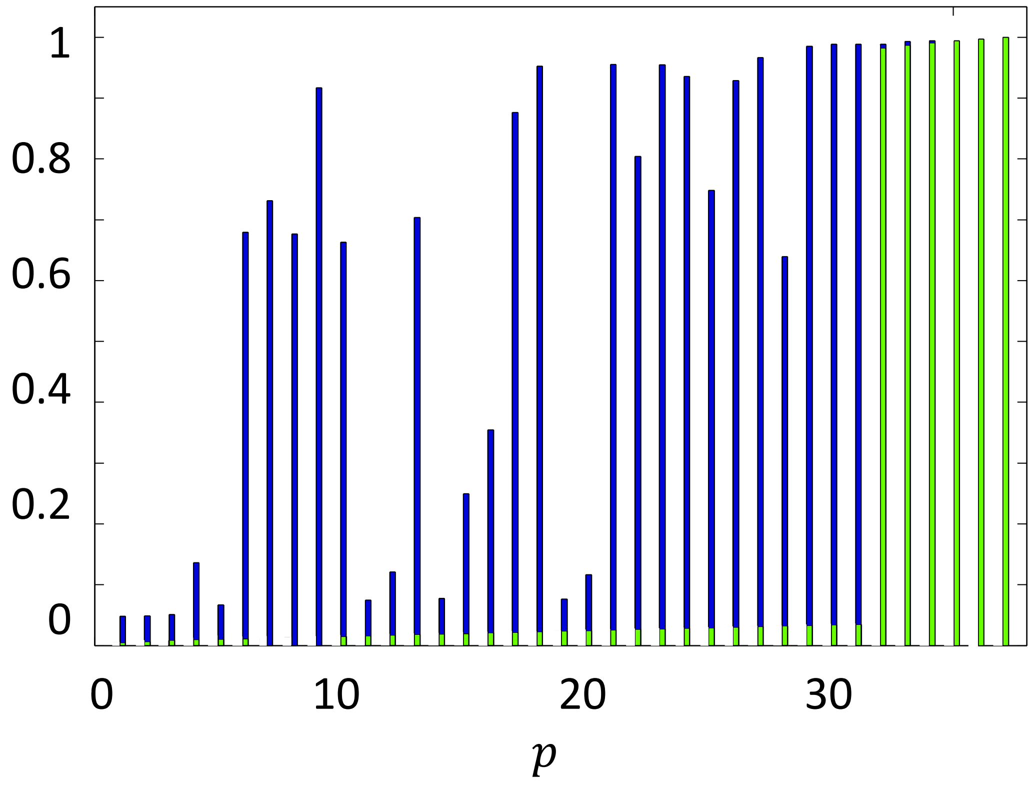
The simulations return the intensity $H(p)$ of the observed informational cascade for each value of the number p of pinned nodes.



Main results

The simulations return the intensity $H(p)$ of the observed informational cascade for each value of the number p of pinned nodes.

We also observe that a fraction of agents reach consensus not only on the trading strategy $\bar{s}$, but also on the opinion of the pinner $\bar{x}$. This corresponds to the fraction of controlled nodes $Q^o(p)$.



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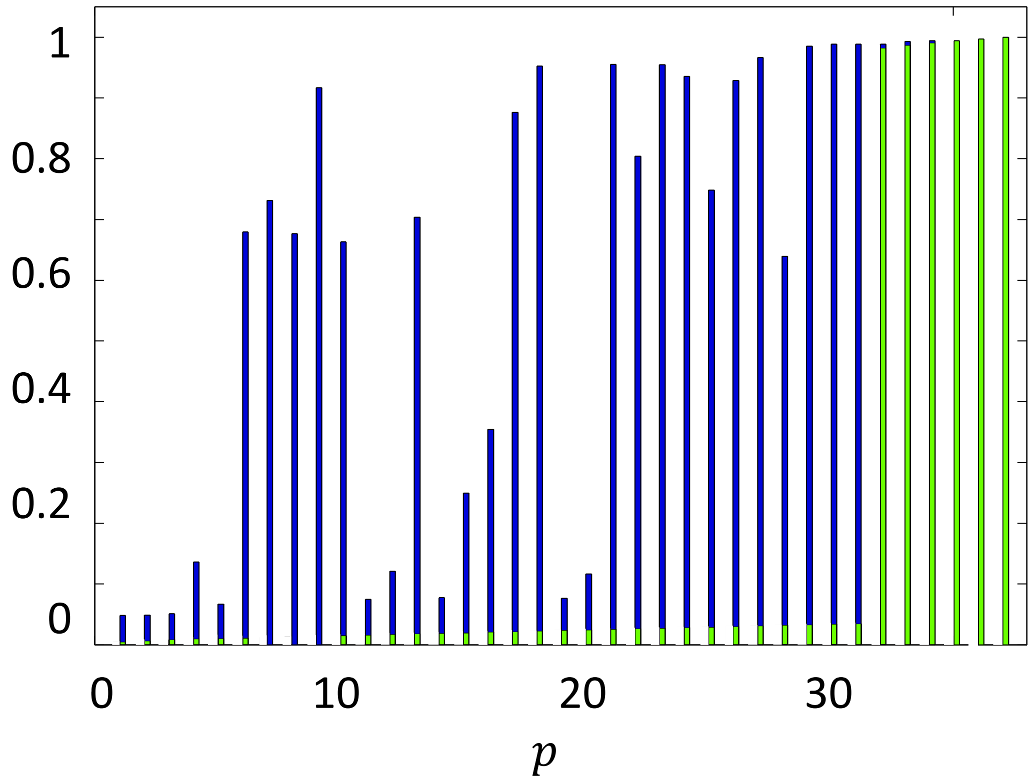
Of course, $Q^o(p) \leq H(p)\ \forall p$, as the strategy is the output, while the opinion is the state.



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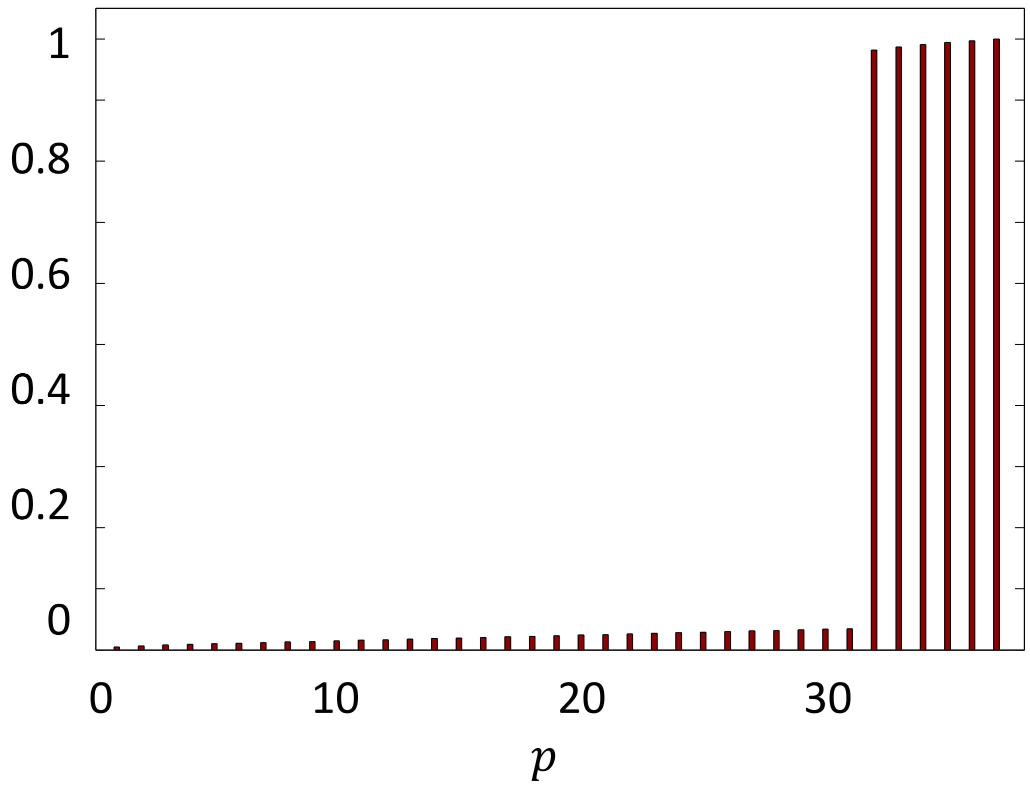
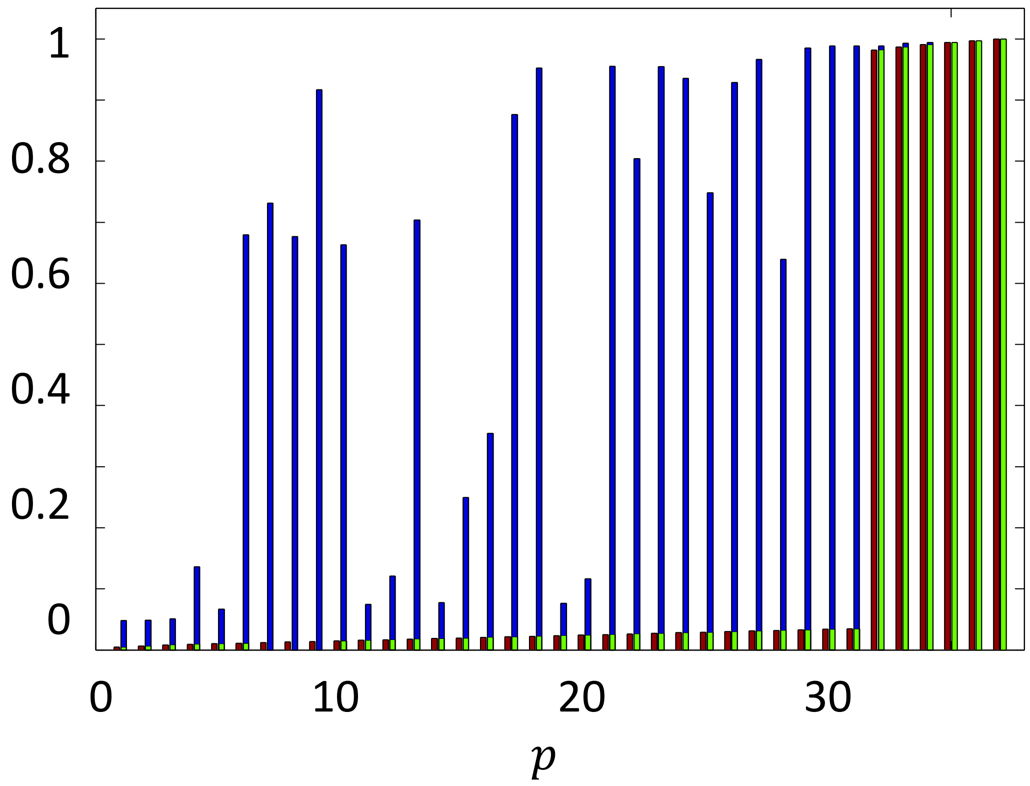
Exploiting the topological conditions of partial pinning control algorithm, we compute the fraction of controllable nodes $Q(p)$ for each number of pinned nodes. This could give us a **prediction** on the intensity of the triggered informational cascade.



Main results: Comparison

The simulations confirm the prediction: the fraction of agents who actually reach consensus on the pinner's opinion $Q^0(p)$ coincides with the fraction of controllable nodes $Q(p)$ returned by the algorithm.



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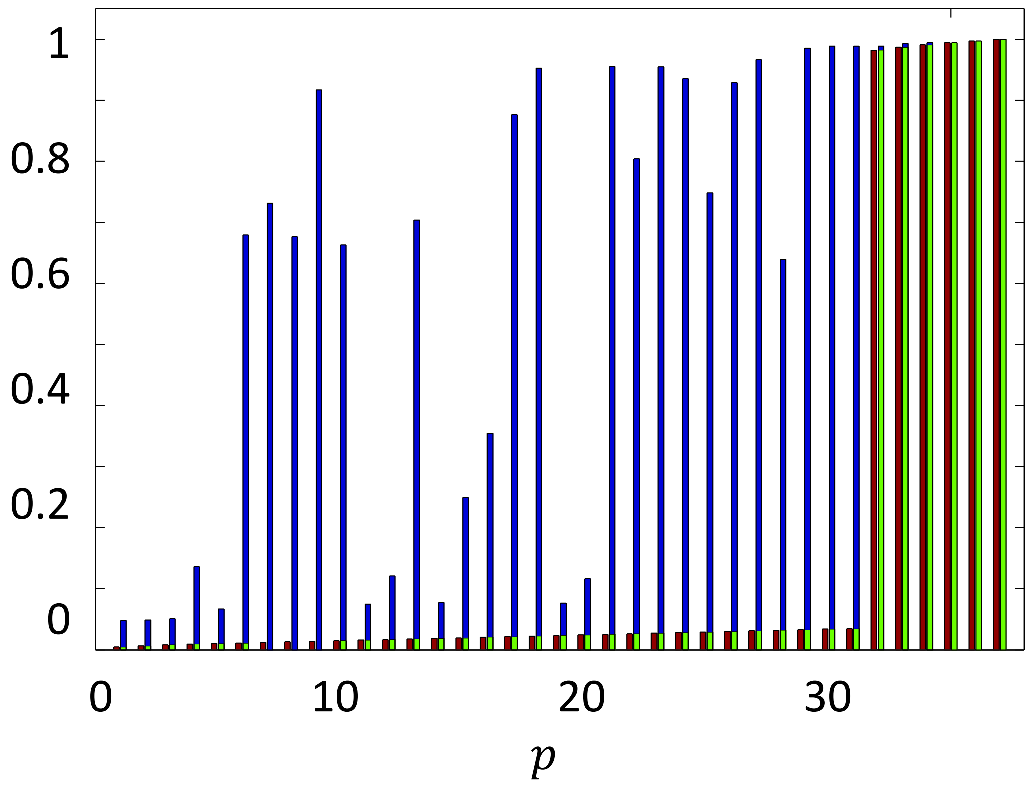

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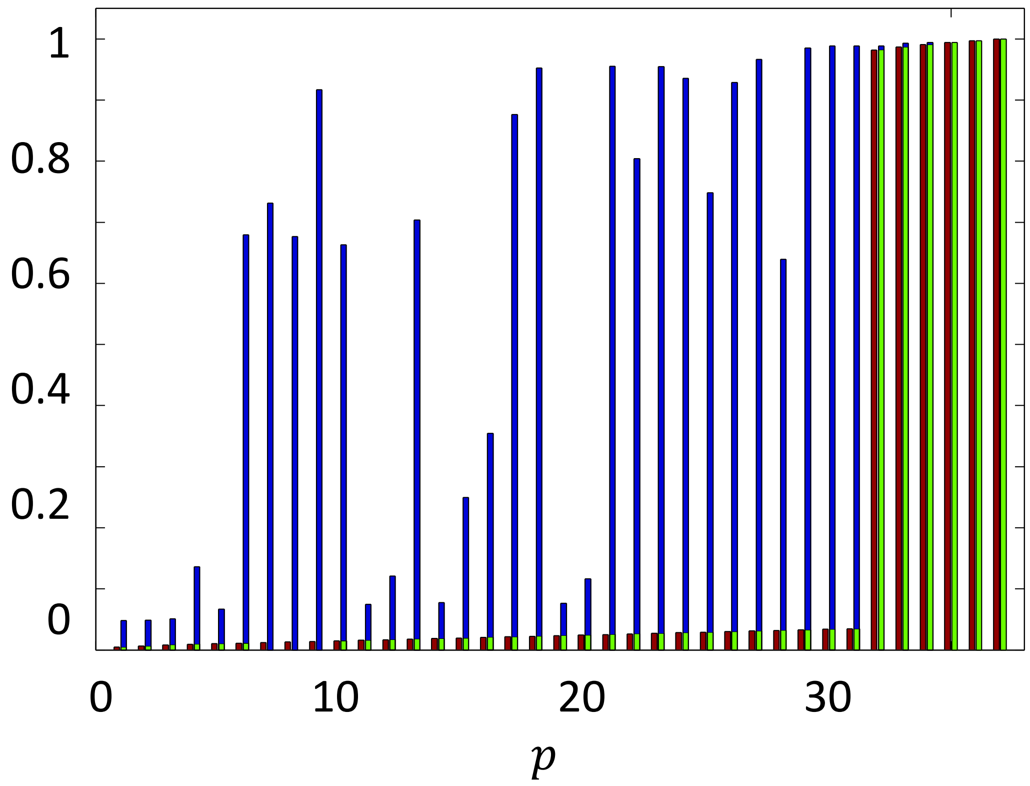
The topological conditions for partial pinning controllability can be actually exploited to make a predition on the minimum intensity of the triggered informational cascade.

Actually, $H(p)$ depends on the output, and not on the state. By varying the output, we could reduce the difference between the actual cascade and the predicted one.



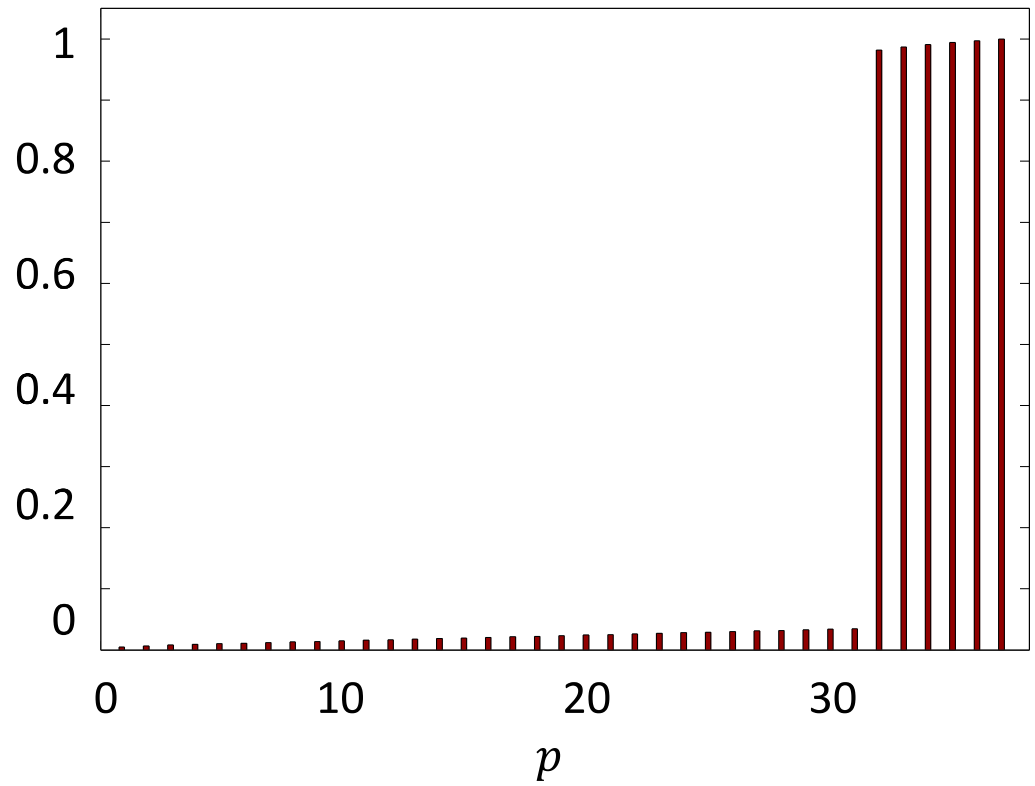
Predictions and the Network Topology

Predictions are made on the basis of the network topology.

What are the **topological features** of a network which influence the intensity of the informational cascade?

Phase Transitions in Partial Pinning Controllability of Complex Networks

We can observe a «*phase transition*» from a fully incoherent behavior to the synchronization of the whole network.



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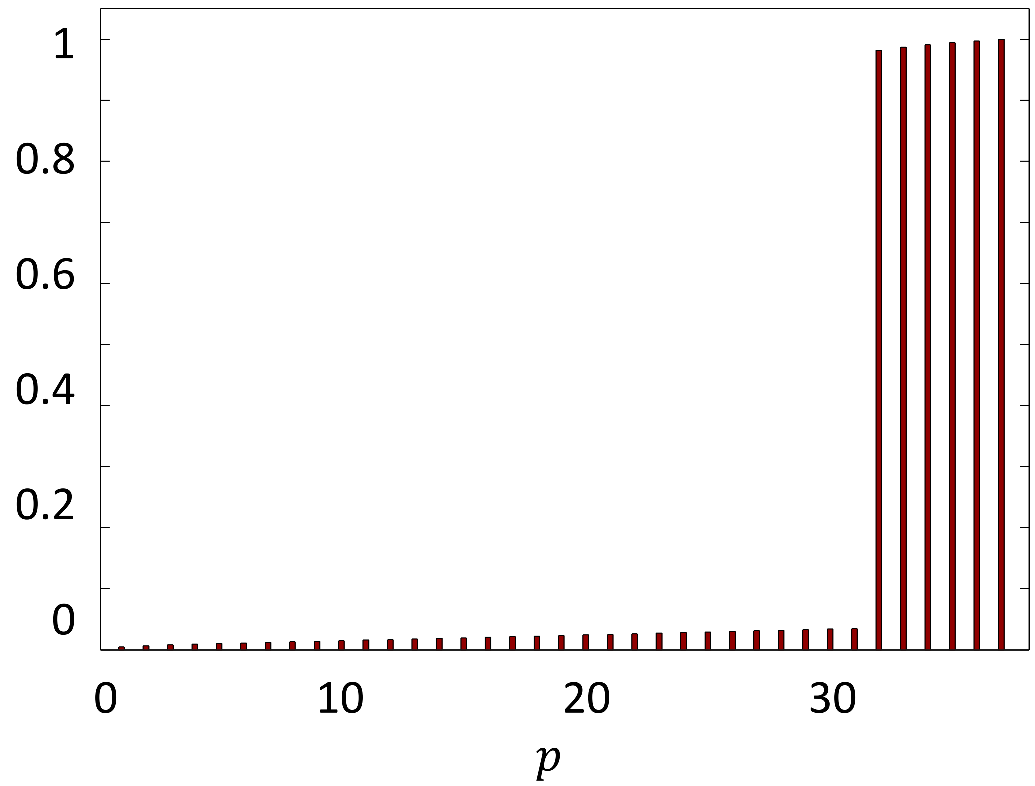

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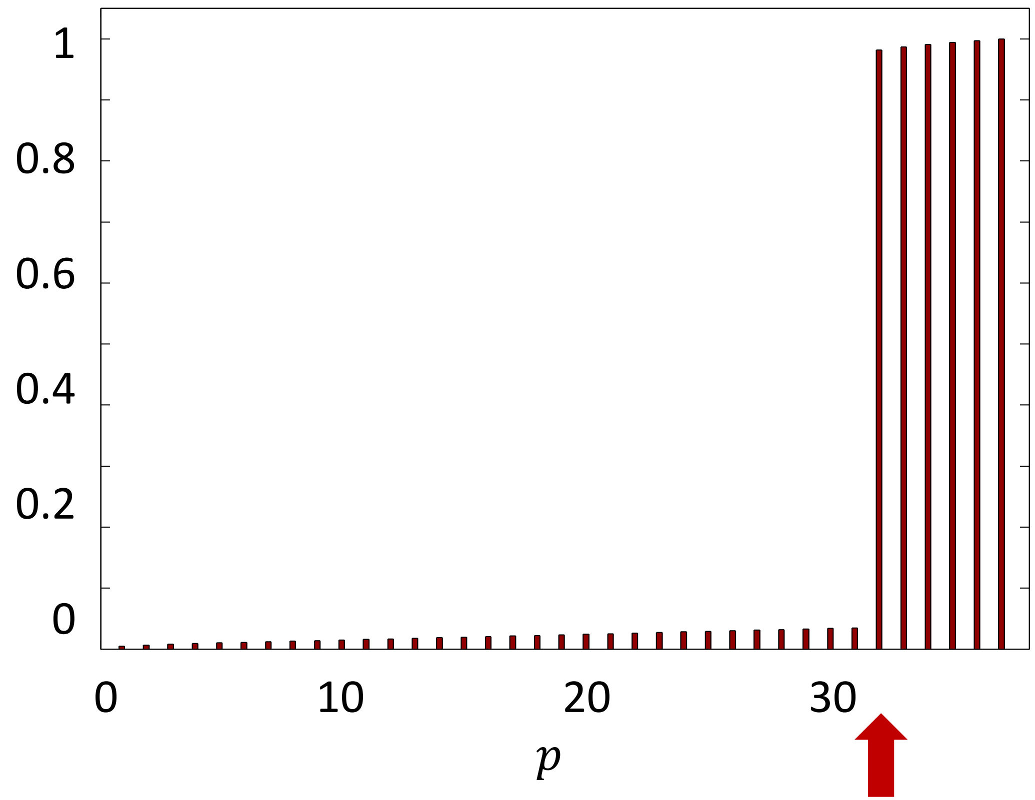
A sufficient condition for the emergence of the *jump* in the transition is the presence of a Giant Strongly Connected Component.



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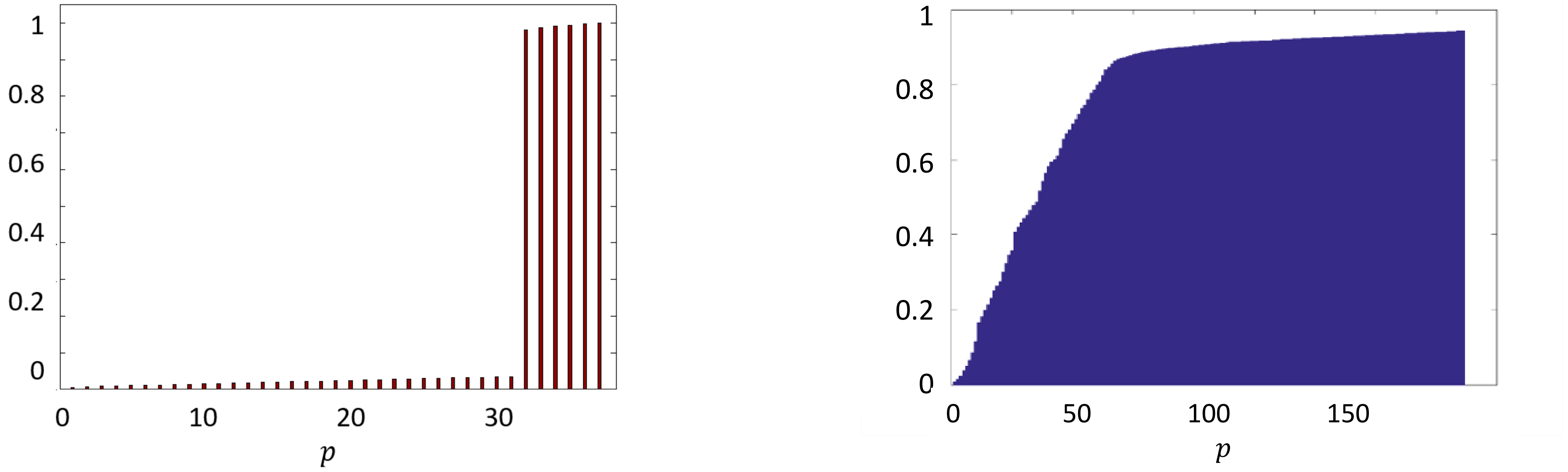

Partial Pinning Controllability of a network with a GSCC (left) and a network without GSCC (right)

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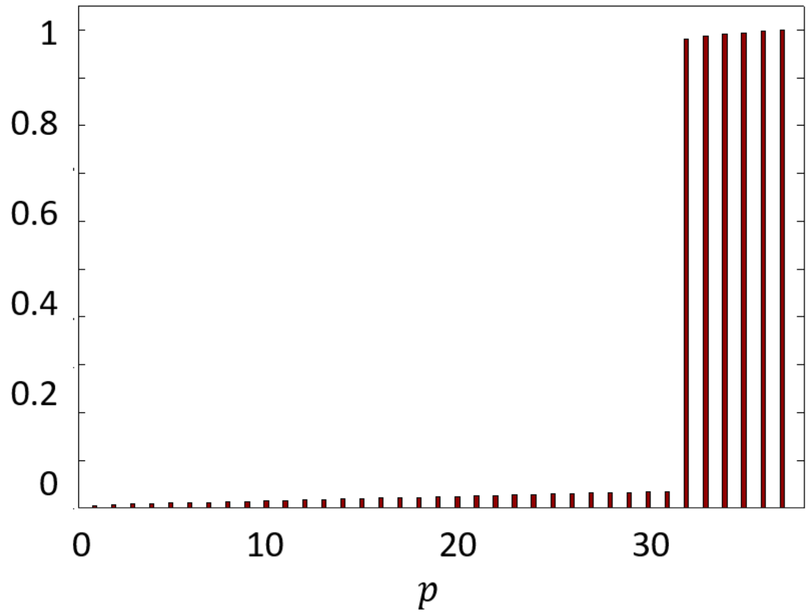

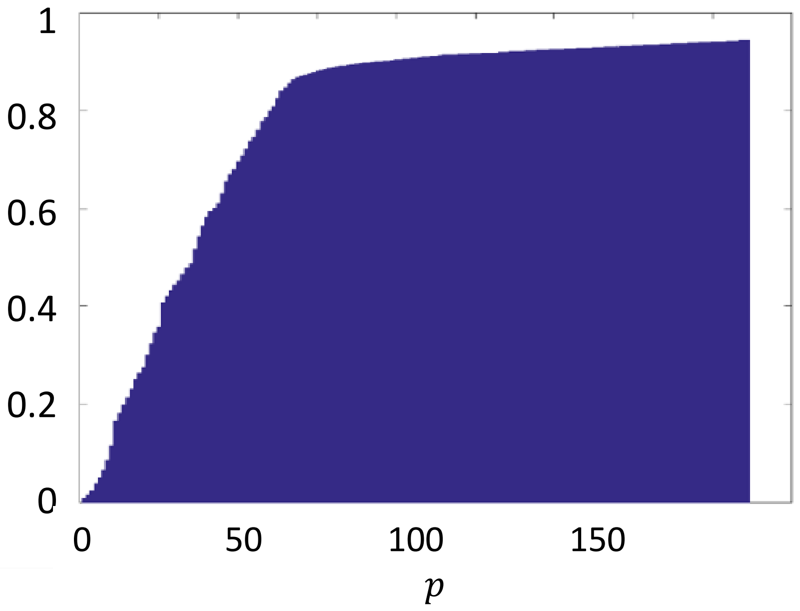

Partial Pinning Controllability of a network with a GSCC (left) and a network without GSCC (right)

The correlation between partial pinning controllability and the topological features of complex networks will be better investigated in future research.

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Our model is capable of replicating informational cascades of **different** and **predictable intensities**.

Exploiting complex networks, we model the influence among the agents taking into account their actual pattern of interactions, and not in a randomic way.

Products

- DeLellis, P., Garofalo, F., Iudice, F. L., & Napoletano, E. (2015). Wealth distribution across communities of adaptive financial agents. *New Journal of Physics*, *17*(8), 083003.

- Garofalo, F., Iudice, F. L., & Napoletano, E. Herding as a consensus problem. *Nonlinear Dynamics (accepted), 2017.*

Student: Elena Napoletano elena.napoletano@unina.it

Tutor: Franco Garofalo franco.garofalo@unina.it

Cycle XXX

	Credits year 1								**Credits year 2**								**Credits year 3**								
		1	2	3	4	5	6			1	2	3	4	5	6			1	2	3	4	5	6		
	Estimated	bimonth	bimonth	bimonth	bimonth	bimonth	bimonth	Summary	Estimated	bimonth	bimonth	bimonth	bimonth	bimonth	bimonth	Summary	Estimated	bimonth	bimonth	bimonth	bimonth	bimonth	bimonth	Summary	Total
Modules	18			3	4	7	6	**20**	9		6		3			**9**								**0**	**29**
Seminars	13		0,8	1,6	0,4		0,2	**3**	6		1,3	0,5	4,6	1,6		**8**					4			**4**	**15**
Research	34	10	9	5	7	6	5	**42**	42	10	3	9	3	8	10	**43**		10	10	10	5	10	10	**55**	**140**
	65	10	9,8	9,6	11	13	11	**65**	57	10	10	9,5	11	9,6	10	**60**	0	10	10	10	9	10	10	**59**	**184**

Thank you!

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