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XXX Cycle - II year presentation

Herding as a consensus problem

MY RESEARCH TOPIC

- Herding (in italian: «raggrupparsi in gregge») includes phenomena such as adaptation, imitation, and learning, which may be caused by unconscious or pre-rational behaviors.
- In financial markets, herding phenomena are quite common and are usually triggered by exogenous information which drags the agents in a blind replication of the same trading decision (the so-called *informational cascades*).
- The mathematical models proposed in the literature of the herding phenomena do not take into account the possibility that not all the agents may herd. However, empirical evidence shows that herding phenomena with different intensities often arise.

THE IDEA

- We view herding phenomena as the convergence of the agents' opinion towards a general **consensus**.
- We introduce a new non linear model of opinion diffusion in the artificial market in [1].
- In our model, each agent is a node of a **dynamical network**.
  His state is his current opinion on the expected value of the asset returns $r_{il}(k)$ and his reputation $x_i(k)$.
  The network edges describe the reciprocal influence among the agents.
- The exogenous information is an external signal injected to a subset of the agents which will play the role of *informed* leaders.
- Our model of opinion diffusion can be seen as a **pinning control problem**, that is capable of triggering herding phenomena of different intensities.
- We look for structural conditions that enable us to predict the number of agents involved in the herding phenomenon reaching consensus to the decision of the informed leaders.

HERDING AS A RESULT OF OPINION CONSENSUS

- *Agent i opinion diffusion equation*

  $r_{il}(k)$ = personal expected return of the l-th asset
  $x_i(k)$ = current wealth

$$\begin{cases} r_{il}(k+1) = r_{il}(k) + \sum_{j \in N_i} \alpha_{ij}(x(k))\left(r_{il}(k) - r_{jl}(k)\right) + \delta_i(\bar{r}_l - r_{il}(k)) \\ x_i(k+1) = f\left(x_i(k), \bar{r}_{l_i^*(k)}, \beta_i(k)\right) \end{cases}$$

  - $i \in I$ = *informed* agent
  - $i \notin I$ = *non informed* agent

$$\alpha_{ij}(x(k)) = \frac{x_j(k)}{\sum_{h \in N_i} x_h(k)}; \quad \delta_i = \begin{cases} 1, i \in I \\ 0, i \notin I \end{cases}$$

- *The herding intensity*

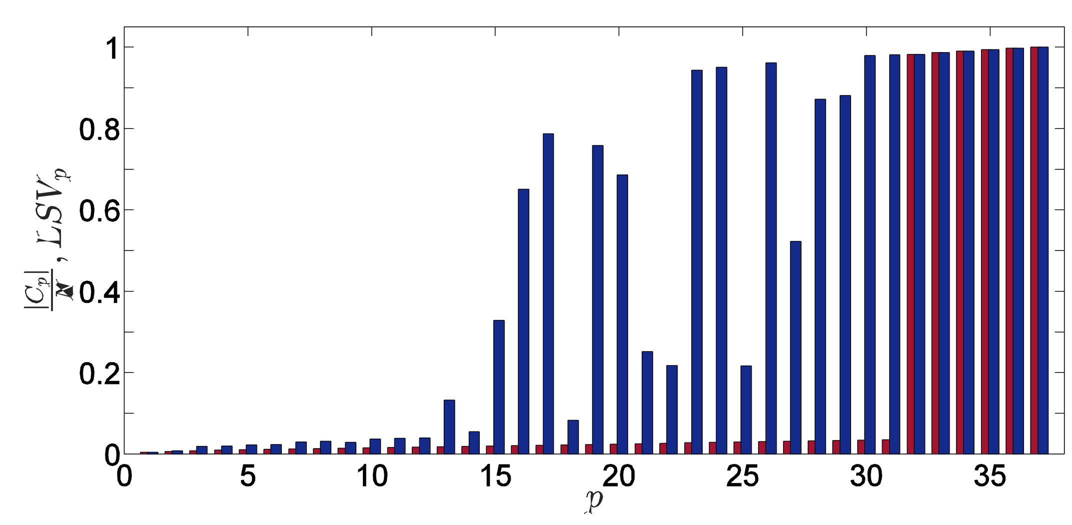
$$LSV_p = \frac{|H_p|}{N}$$

  H_p is the set of agents which uniform their trading strategies to that of the pinner.

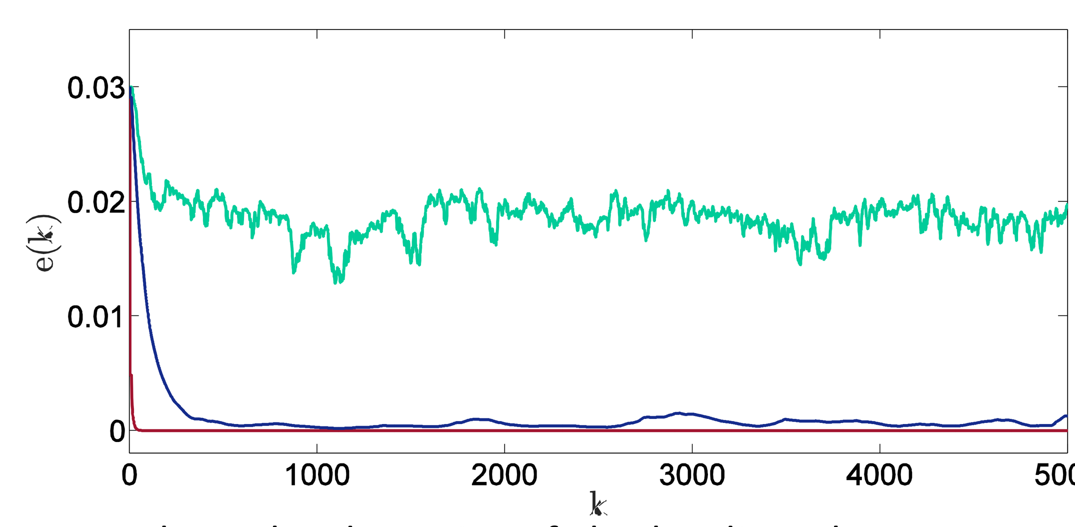
- *The trading decision*

$$l_i^*(k) = \arg\max_{l \in L_i(k)} d_i(k) r_{il}(k)$$

NUMERICAL RESULTS AND COMPARISON WITH THE PREDICTIONS OF PARTIAL PINNING



In this picture, p is the number of pinned nodes. C_p is the maximal set of potential followers of the pinned nodes as evaluated by the *partial pinning* algorithm in [2] and $|C_p|$ its cardinality. The red bars represent the fraction of potential followers with respect to the total number of agents in the network. The blue bars represent the LSV_p index evaluated by numerical simulations.
The numerical results confirm the predictions of *partial pinning*, as $C_p \subseteq H_p \; \forall p$.



In this picture, we show the dynamics of the herding phenomenon. The red and blue lines represent the absolute mean error between the opinion of the pinner (the leader) and that of the agents belonging to C_p and H_p, respectively. The green line represent the absolute mean error of the remaining agents.

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Nonlinear Systems, Networks and Control

Future works
We aim at studying pinning controllability on different kinds of artificial and real networks, to point out possible correlations among network topologies, characteristics of pinned nodes and the spread of information in the market in terms of herding intensity.

References

[1] DeLellis, P., Garofalo, F., Iudice, F. L., & Napoletano, E. (2015). Wealth distribution across communities of adaptive financial agents. *New Journal of Physics*, *17*(8), 083003.
[2] DeLellis, P., Garofalo, F., & Iudice, F. L. The partial pinning control strategy for large complex networks. Accepted for presentation at the *55th IEEE Conference on Decision and Control*, 12-14 December 2016.