Nicola Isernia

Tutor: Fabio Villone

XXXIV Cycle - III year presentation

The Electromagnetic Interaction of Magneto-Hydro-Dynamic Plasmas with Conductors

Università degli Studi di Napoli Federico II

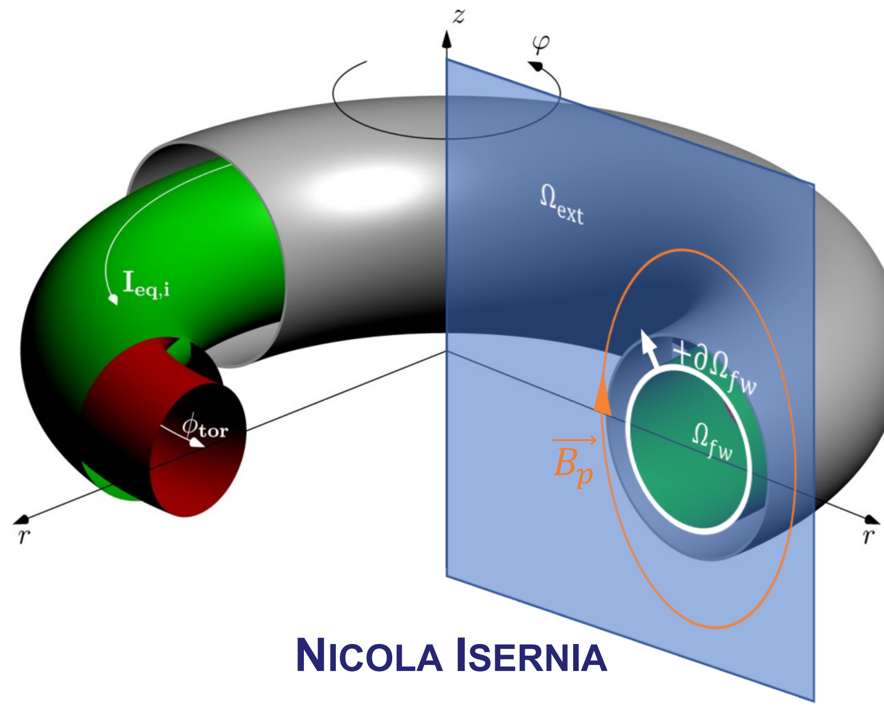
Abstract The tight coupling between the macroscopic evolution of Tokamak plasmas and the induced currents in the surrounding Vacuum Vessel (VV) and Plasma Facing Components (PFCs) has been known for decades. In the present Thesis we critically review some aspects of the electromagnetic interaction. In conditions of significant plasma-wall contact the gas mixture is generally only *partially* ionized. We try to model this situation in a consistent thermodynamic framework, allowing for ionization and the recombination phenomena, in Chapter 1. This represents the occasion to review the whole MHD theory in the wider framework of Non-equilibrium Thermodynamics, allowing to discuss the implications of the *Curie* principle on the closure relations generally adopted. A self-consistent coupling of 3D non-linear MHD models with fully volumetric 3D structures models is still missing in the literature. We explore some possibilities in Chapter 2, hinting also the first preliminary results in the JOREK-CARIDDI coupling. Several possible formulations are discussed, together with the possible implications of halo currents in the modeling. In Chapter 3 we discuss the mass-less hypothesis and the fundamental aspects of MHD evolutionary equilibrium models. Here we also review the key aspects of the numerical model *CarMa0NL*. In the last Chapter we apply the *evolutionary equilibrium tools* previously discussed to practical problems. We first successfully cross-check analytical and numerical computation of forces during off-normal events called disruptions, providing some hints on the magnetic tensions, besides on the magnetic pressures. Further, we propose a procedure for the estimation of plasma losses during disruptions via evolutionary equilibrium models, which we apply to a simple test case. We find also in this case the fundamental role of the electromagnetic time constant, which regulates the plasma dissipated heat during the current quench phase. Further we validate *CarMa0NL* by direct comparison with JET and TCV experiments, comparing simulated and real magnetic diagnostics measurements. For JET, we find that the halo width is a crucial element for a realistic simulation. In the TCV studies we show that the disruption trajectory is dependent on the pre-disruption plasma shape.

Nicola Isernia

Electromagnetic interaction of MHD plasmas with structures. 2021 - XXXIV

UNIVERSITÀ DEGLI STUDI DI NAPOLI FEDERICO II

THE ELECTROMAGNETIC INTERACTION OF MAGNETO-HYDRO-DYNAMIC PLASMAS WITH CONDUCTING STRUCTURES



NICOLA ISERNIA

PH.D. IN INFORMATION TECHNOLOGY AND ELECTRICAL ENGINEERING

Background

- M.Sc. In Electrical Engineering (*Federico II, March 2018)*
- *Athenaeum Fellowship*
- Research group: *Electrical Engineering* & *Consorzio CREATE*
- This year collaborations:

Max Planck Institute for Plasma Physics

CCFE CULHAM CENTRE FOR FUSION ENERGY

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE | SWISS PLASMA CENTER

IPP INSTITUTE OF PLASMA PHYSICS OF THE CZECH ACADEMY OF SCIENCES

Background

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- This year collaborations:

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CCFE ** CULHAM CENTRE FOR FUSION ENERGY

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IPP INSTITUTE OF PLASMA PHYSICS OF THE CZECH ACADEMY OF SCIENCES ***

* Plasma-Conductors models Interaction (3 months research abroad in Garching)
** Interpretation of Experiments via simulations
*** Modelling of forces for next tokamak COMPASS-U

itee Ph.D INFORMATION TECHNOLOGY ELECTRICAL ENGINEERING

Background - List of Publications

International journal papers

V.V. Yanovskiy, N. Isernia, V.D. Pustovitov, F. Villone, D. Abate, P. Bettini, S.L. Chen, J. Havlicek, A. Herrmann, J. Hromadka, M. Hron, M. Imrisek, M. Komm, R. Paccagnella, R. Panek, G. Pautasso, S. Peruzzo, D. Sestak, M. Teschke, I. Zammuto, "*Comparison of approaches to the electromagnetic analysis of COMPASS-U vacuum vessel during fast transients"*, Fusion Engineering and Design, Volume 146, Part B, pp. 2338-2342, sep 2019, DOI: 10.1016/j.fusengdes.2019.03.185

N. Isernia, V. D. Pustovitov, F. Villone,V. Yanovskiy, "*Cross-validation of analytical models for computation of disruption forces in tokamaks*", Plasma Physics and Controlled Fusion, Volume 61, Number 11, pp. 115003, sep 2019, DOI: 10.1088/1361-6587/ab4016

N. Isernia, V. Scalera, C. Serpico, F. Villone, "Energy balance during disruptions", Plasma Physics and Controlled Fusion, Vol. 62 (9), pp. 095024, aug 2020, DOI: 10.1088/1361-6587/ab9074

V.V. Yanovskiy, N. Isernia, V. D. Pustovitov, V. Scalera, F. Villone, J. Hromadka; M. Imrisek, J. Havlicek, M. Hron, and R. Panek "*Global forces on the COMPASS-U wall during plasma disruptions*", Nuclear Fusion, Vol. 61 (9), pp. 096016, aug 2021, DOI: 10.1088/1741-4326/ac1545

P. Vondracek, R. Panek, M. Hron, J. Havlicek, V. Weinzettl, T. Todd, D. Tskhakaya, G. Cunningham, P. Hacek, J. Hromadka, P. Junek, J. Krbec, N. Patel, D. Sestak, J. Varju, J. Adamek, M. Balazsova, V. Balner, P. Barton, J. Bielecki, P. Bilkova, J. Błocki, D. Bocian, K. Bogar, O. Bogar, P. Boocz, I. Borodkina, A. Brooks, P. Bohm, J. Burant, A. Casolari, J. Cavalier, P. Chappuis, R. Dejarnac, M. Dimitrova, M. Dudak, I. Duran, R. Ellis, S. Entler, J. Fang, M. Farnik, O. Ficker, D. Fridrich, S. Fukova, J. Gerardin, I. Hanak, A. Havranek, A. Herrmann, J. Horacek, O. Hronova, M. Imrisek, N. Isernia, F. Jaulmes, M. Jerab, V. Kindl, M. Komm, K. Kovarik, M. Kral, L. Kripner, E. Macusova, T. Majer, T. Markovic, E. Matveeva, K. Mikszuta-Michalik, M. Mohelnik, I. Mysiura, D. Naydenkova, I. Nemec, R. Ortwein, K. Patocka, M. Peterka, A. Podolnik, F. Prochazka, J. Prevratil, J. Reboun, V. Scalera, M. Scholz, J. Svoboda, J. Swierblewski, M. Sos, M. Tadros, P. Titus, M. Tomes, A. Torres, G. Tracz, P. Turjanica, M. Varavin, V. Veselovsky, F. Villone, P. Wąchal, V. Yanovskiy, G. Zadvitskiy, J. Zajac, A. Zak, D. Zaloga, J. Zelda, H. Zhang, "Preliminary design of the COMPASS upgrade tokamak", Fusion Engineering and Design, Volume 169, 2021,112490, ISSN 0920-3796, DOI: 10.1016/j.fusengdes.2021.112490

S. Perna, V. Scalera, M. d'Aquino, N. Isernia, F. Villone and C. Serpico, "Magnetostatic Field Computation in Thin Films Based on k-Space Fast Convolution With Truncated Green's Function" in IEEE Transactions on Magnetics, vol. 58, no. 2, pp. 1-6, Feb. 2022, Art no. 7000106, doi: 10.1109/TMAG.2021.3079474.

Background - List of Publications

International Conference papers

N. Isernia, V. Scalera, C. Serpico, F. Villone, “Energy balance during disruptions”, 46th Plasma Physics Conference of the European Physical Society, Milan, 2019. (OCS: P4.1053)

S. Chen, F. Villone, Y. Sun, B. Xiao, N. Isernia, G. Rubinacci, S. Ventre, “Simulation of disruptions in EAST tokamak”, 46th Plasma Physics Conference of the European Physical Society, Milan, 2019 (OCS: O4.110)

S. Jardin, F. Villone, C. Clauser, N. Ferraro, N. Isernia, G. Rubinacci, S. Ventre, “ITER disruption simulations with realistic plasma and conductors modelling”, 46th Plasma Physics Conference of the European Physical Society, Milan, 2019 (OCS: P5.1003)

V. Yanovskiy, N. Isernia, V.D. Pustovitov, F. Villone, J. Havlicek,A. Havranek, J. Hromadka, M. Hron, F. Jaulmes, M. Komm, O. Kovanda, K. Kovarik, J. Krbec, T. Markovic, E. Matveeva, R. Panek, J. Seidl, D. Tskhakaya, V. Weinzettl, “Poloidal currents in COMPASS vacuum vessel during symmetrical disruptions: measurements using diamagnetic loop and comparison with CarMa0NL modelling”, 46th Plasma Physics Conference of the European Physical Society, Milan, 2019 (OCS: P4.1056)

M. Cianciosa, N. Isernia, G. Rubinacci, D. Terranova, F. Villone, “Coupled Modeling for Self Consistent Equilibrium Evolution Using the IPS Framework”, 46th Plasma Physics Conference of the European Physical Society, online, 2021, (OCS: P1.1038)

F. Villone, S. Coda, N. Isernia, G. Rubinacci, the TCV and EUROFusion MST1 Team, “Disruption trajectory studies on TCV: experiments and modelling”, 47th EPS Conference on Plasma Physics, online, 2021 (P4.1029)

iNFORMATION tECHNOLOGY eLECTRICAL eNGINEERING Ph.D

Credits Summary

- This Year:
  - 1 Module: *Introduzione ai Circuiti Quantistici* (9)
  - Several Seminars (ITEE, SSM, International PhD in Fusion Science and Technology, …)

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Tutor: Fabio Villone
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Cycle XXXIV

	Credits year 1								Credits year 2								Credits year 3									
	Estimated	bimonth 1	bimonth 2	bimonth 3	bimonth 4	bimonth 5	bimonth 6	Summary	Estimated	bimonth 1	bimonth 2	bimonth 3	bimonth 4	bimonth 5	bimonth 6	Summary	Estimated	bimonth 1	bimonth 2	bimonth 3	bimonth 4	bimonth 5	bimonth 6 + ext	Summary	Total	Check
Modules	23	9	9.4		9			**27**	15		3			8.4	4	**15**	20				9			**9**	**51.8**	30-70
Seminars	7	0	0.6	0.7	1	0	3	**5.3**	3	1.4		0.8	0.5		1.5	**4.2**	5	1.1	0.4	0.6	0.8		0.2	**3**	**12.6**	10-30
Research	34	2	1	6.3	5	8	6	**28**	42	8.6	7	9.2	9.5	1.6	4.5	**40**	34	8.6	8.6	8	4	8.7	24	**62**	**131**	80-140
	64	11	11	7	15	8	9	**61**	60	10	10	10	10	10	10	**60**	59	9.7	9	8.6	14	8.7	24	**74**	**195**	180

Fusion: the nuclear reaction that powers the sun

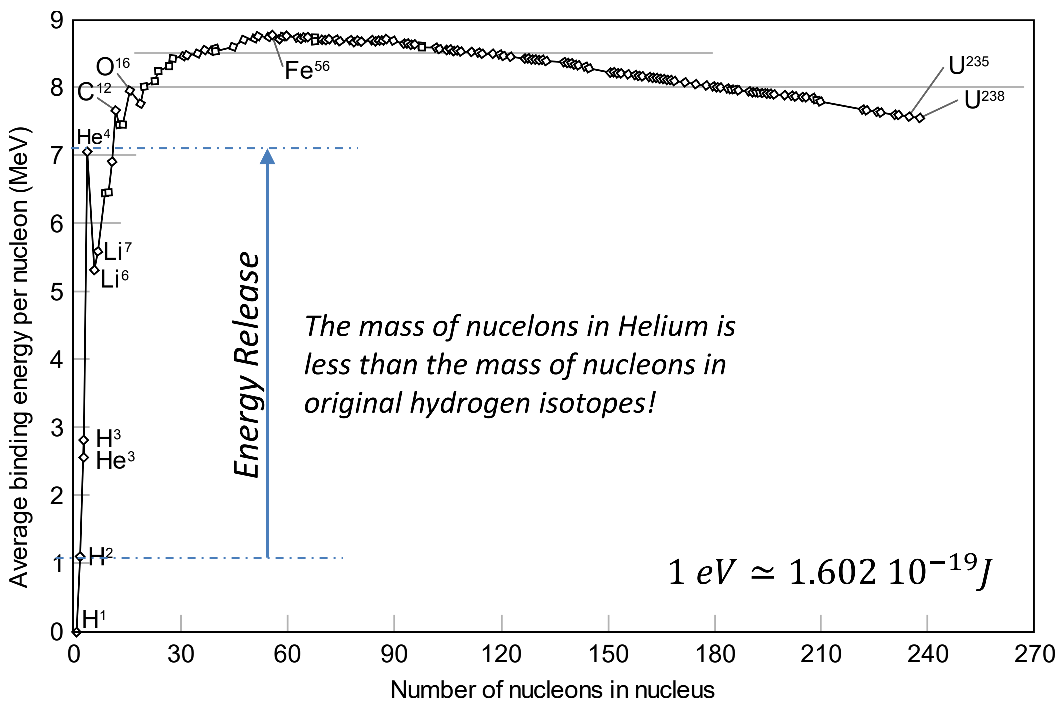
Standard Methane Combustion

$$CH_4 + 2\,O_2 = CO_2 + 2\,H_2O + \mathbf{8.4\ eV}$$

Deuterium-Tritium Fusion reaction

$$^2H + \ ^3H = \ ^4He + n + \mathbf{17.6\ MeV}$$

Fuel	MJ/kg
D-D	78 10^6
D-T	338 10^6
CH_4	40



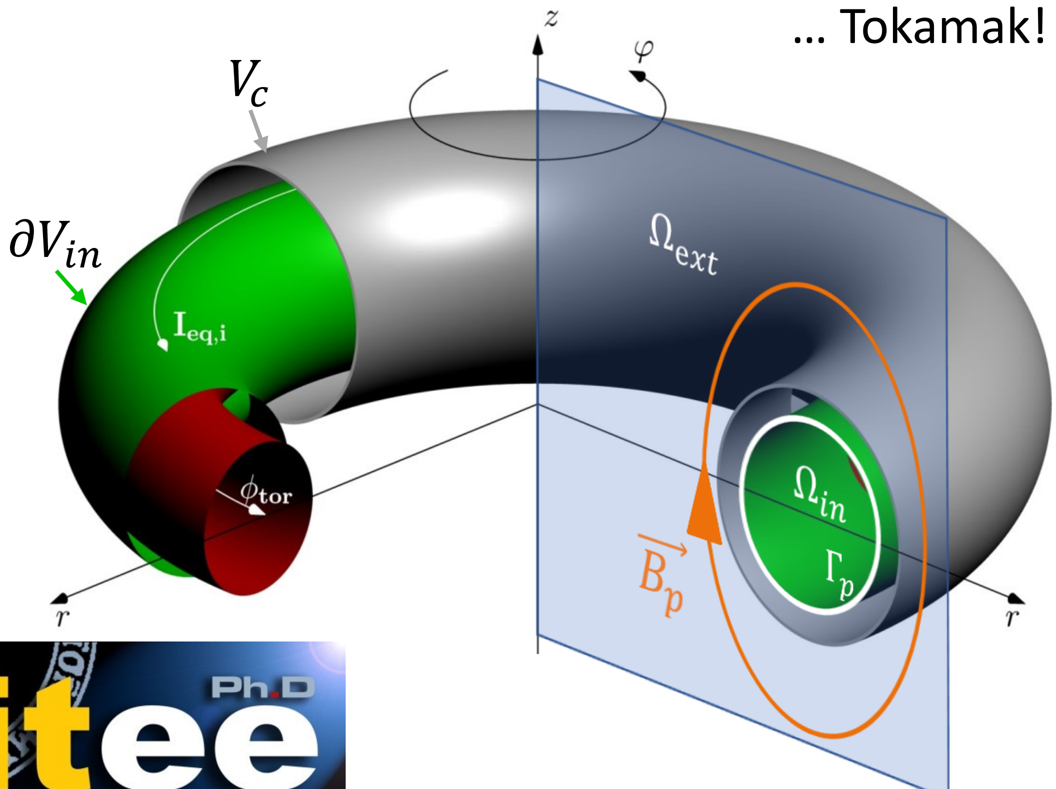
- Large availability of Deuterium in see water
- Sustainability: Breeding Tritium from Lithium
- Intrinsic safety, fusion reactions are never spontaneous
- *Fusion Energy* is the energy of stars. **How can we do that on earth?**

«Building today the energy of tomorrow»

- Need to reach 10 times the sun core temperature
- Lack of gravitational confinement **but** locally neutral, highly conducting ionized gas (e.g. $\eta_\perp \simeq [10^{-8},\ 10^{-10}]\ \Omega \cdot m$, $\eta_\parallel \simeq \eta_\perp/2$)
- Force balance…

$$\boldsymbol{i} \times \boldsymbol{B} = \nabla p$$

… Tokamak!



Example: JET M18-33	
I_p	2 MA
B_φ	2 T
F_z	$100\ tons$
T_e	$17 \cdot 10^6\ K$

- A current can be induced within the gas with no plasma-wall contact
- Concept by the Soviet physicsts I. Tamm and A. Sakharov in the '50s
- Declassification of Nuclear research later in 1958 (2° IAEA Conference on the Peaceful uses of Fusion Energy)

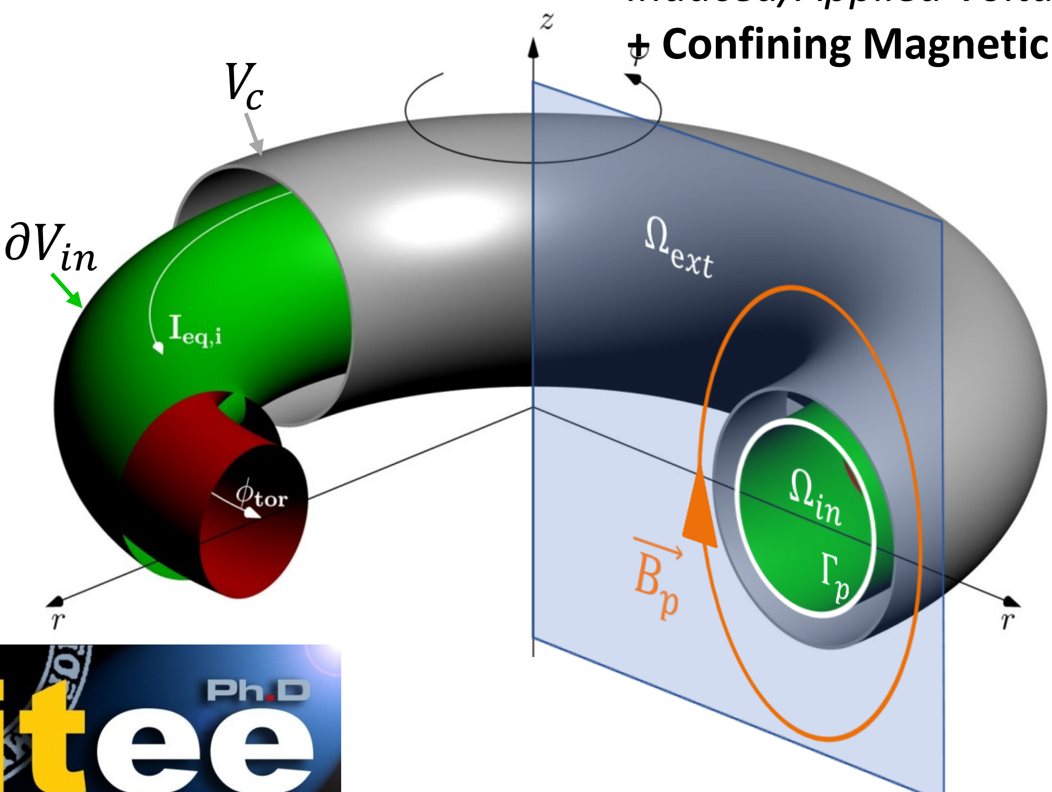
The electromagnetic Interaction

Plasma current variations:
- *Distribution*
- *Position*
- *Shape*

Induced/Applied Voltages →

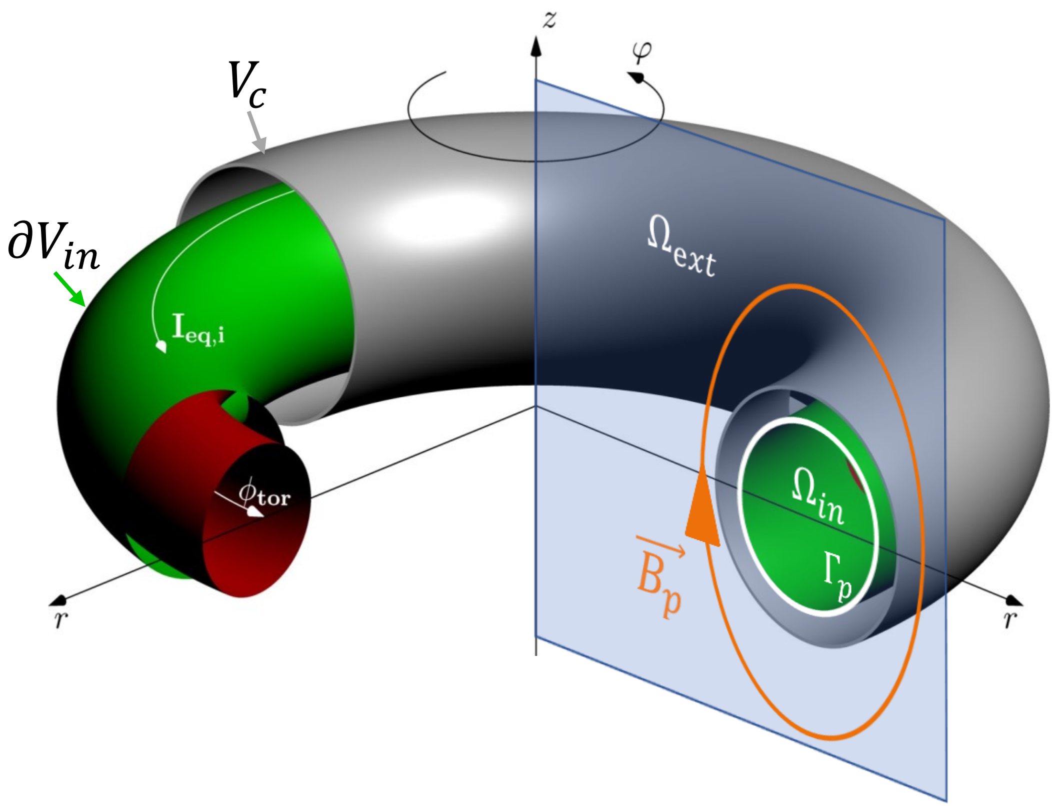
← *Induced/Applied Voltages* **+ Confining Magnetic field!**

External currents variations
- *Induced currents*
- *Injected currents/Applied Voltages*



Relevance:
- Plasma macroscopic motion;
- Mechanical stability of the plasma column;
- Electromagnetic forces on surrounding structures;
- Energy fluxes between plasma and external environment

The electromagnetic Interaction



Outline of this talk:

1. **MHD: an irreversible thermodynamics application**
   (*a model for a reacting fluid conductor*)

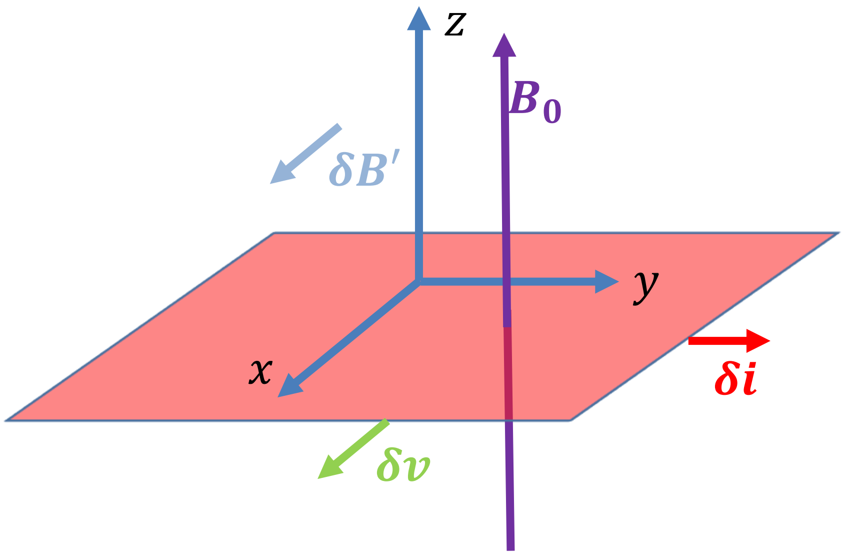
2. **The interaction of extended MHD and MQS conductors models**
   (*a fully 3D tool for studying plasma macroscopic dynamics*)

3. **Experimental validation of evolutionary equilibrium models**
   (the study of plasma motion in the mass-less hypothesis)

Magneto-Hydro-Dynamics

A theory for fluid conductors

1942 *H. Alfvèn* «Existence of electromagnetic-hydrodynamic waves», Nature, Vol. 150, 405-406



$$\begin{cases} \rho \dfrac{d}{dt}\delta\boldsymbol{v} = \mathrm{i} \times \boldsymbol{B} \\ \nabla \times \boldsymbol{B} = \mu_0 \mathrm{i} \\ \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = -\dfrac{\partial}{\partial t}\boldsymbol{B} \end{cases}$$

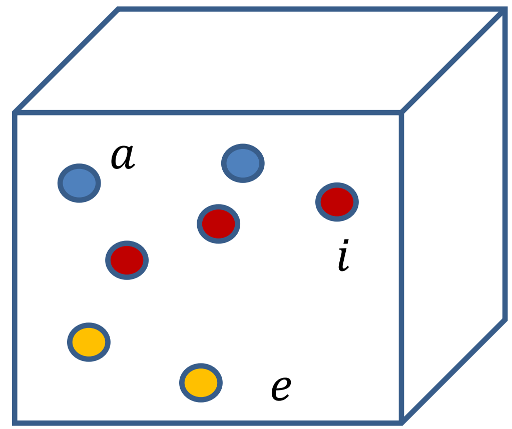
$\eta \simeq 0$

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \eta \boldsymbol{i}$$

$$\left(\frac{B_0}{\sqrt{\mu_0 \rho_0}}\right)^2 \frac{\partial^2}{\partial z^2}\delta B - \frac{\partial^2}{\partial t^2}\delta B = 0$$

Alfvèn velocity

1949 *H. Grad «On the kinetic theory of rarified gases»*, Communications in Pure and Applied Mathematics, Vol.2 (4), pp. 331-407



- *Boltzmann Equation* (Indistinguishable particles + molecular chaos)

$$\frac{\partial f_\alpha}{\partial t} + \dot{\boldsymbol{q}}_{\boldsymbol{\alpha}} \cdot \frac{\partial f_\alpha}{\partial \boldsymbol{q}_{\boldsymbol{\alpha}}} + (e_\alpha \boldsymbol{E} + \dot{\boldsymbol{q}}_{\boldsymbol{\alpha}} \times \boldsymbol{B}) \cdot \frac{\partial f_\alpha}{\partial \boldsymbol{p}_{\boldsymbol{\alpha}}} = \left.\frac{\partial f_\alpha}{\partial t}\right|_{\boldsymbol{c}}$$

Expansion in Hermite polynomials of the velocity

- *Fluid variables* and *conservation laws*

Magneto-Hydro-Dynamics

An Irreversible Thermodynamics Application

1952 *J.G. Kirkwood* and *Jr. B. Crawford* «The macrosocpic equations of transport», Journal of Physical Chemistry, Vol. 56 (9), pp. 1048-1051

1962 *S.R. de Groot and P. Mazur* «Non-equilibrium Thermodynamics», *North-Holland Publishing Company*

- Local or *constrained thermodynamic equilibrium*
- First principles conservation laws for fluid variables
- Consistent non-conservation law for the entropy
- Identification of *thermodynamic fluxes* and *forces*
- ***Entropy** and **symmetry** consistent phenomenological closure of the mathematical model*

Motivations

- *Inclusion of ionization/recombination reactions in the fluid model*
- *Understanding of the closure relations adopted in classical MHD models*
- *Individuation of constraints on the closure relations*

Magneto-Hydro-Dynamics

An Irreversible Thermodynamics Application

- The non-conservation of entropy

$$\rho \frac{\mathrm{d}s}{\mathrm{d}t} = -\nabla \cdot \boldsymbol{K_s} + \sigma$$

where the entropy current density is given by

$$\boldsymbol{K_s} = \frac{\boldsymbol{K_q^*}}{T} + s_p^* \, \boldsymbol{i}^* + s_a^* \, \boldsymbol{j_a}$$

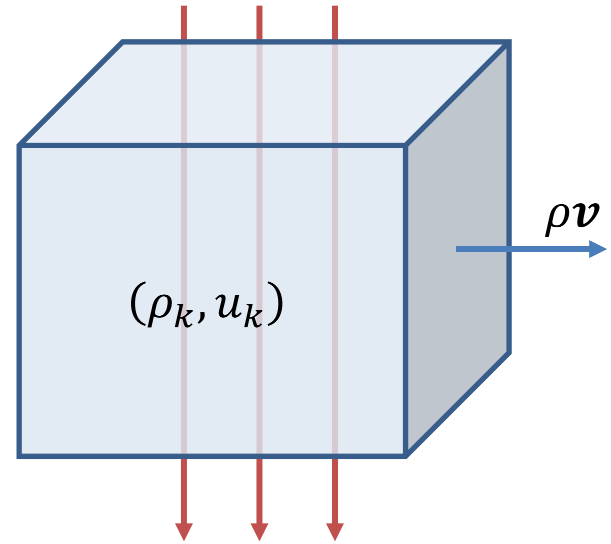
$$\boldsymbol{K_q^*} = \boldsymbol{K_q} - h_p^* \, \boldsymbol{i}^* - h_a^* \boldsymbol{j_a}$$

and the **entropy production** term is

$$\sigma = \sigma_{odd} + \sigma_{even}$$

$$\sigma_{even} = -\frac{1}{T} \underline{\Pi} : \nabla \boldsymbol{v} - \frac{1}{T} J_r A_r \quad \text{where} \quad A_r = \sum_j \nu_j \mu_j$$

$$\sigma_{odd} = \frac{\boldsymbol{K_q^*}}{T} \cdot \left(-\frac{\nabla T}{T} \right) + \frac{\boldsymbol{j_a}}{T} \cdot (-\nabla_T \, \mu_a^*) + \frac{\boldsymbol{i}^*}{T} \cdot \left[-\nabla_T \, \mu_p^* + \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \right]$$



Magneto-Hydro-Dynamics

An Irreversible Thermodynamics Application

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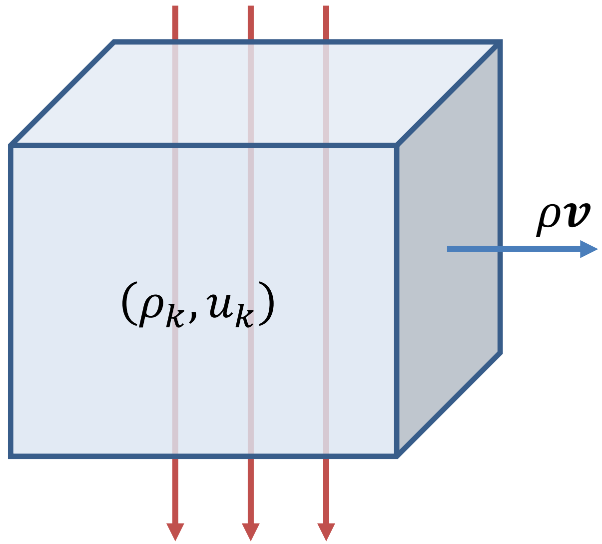


Linear *constitutive equations → entropy production =* ***bilinear*** *form*

$$\left[\underline{\Pi}, J_r, \boldsymbol{K_q^*}, \boldsymbol{j_a}, \boldsymbol{i}^* \right] = \mathcal{L}\left(\nabla \boldsymbol{v}, A_r, -\nabla T / T, -\nabla_T \mu_p^* + \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \right)$$

Magneto-Hydro-Dynamics

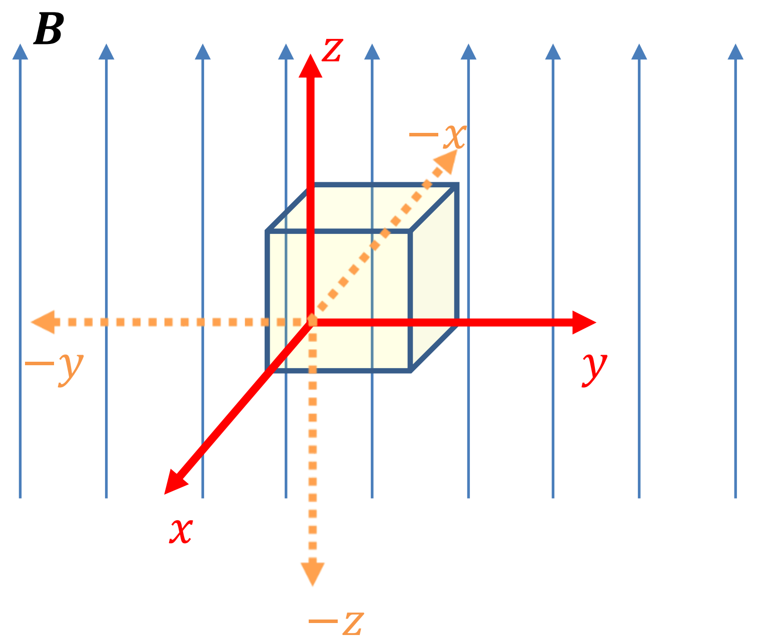
An Irreversible Thermodynamics Application

- The magnetic field is a two-form field (*i.e. 2nd order skew-symmetric tensor field)*

$$\tilde{B} \triangleq \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix} \overset{\varpi}{\Rightarrow} \boldsymbol{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

- It is the only responsible for the anisotropy of our space, *i.e.* $\tilde{B}$ is the structural tensor for our symmetry group
- Isotropization Theorem:

$$\begin{bmatrix} \text{The set } \{\tilde{B}\} \\ characterizes \\ the\ symmetry \end{bmatrix} \Rightarrow \left[T(V) = T_{ISO}\left(V, \tilde{B}\right)\right]$$



The material inversion allows to discard any coupling between even and odd order fluxes and forces!

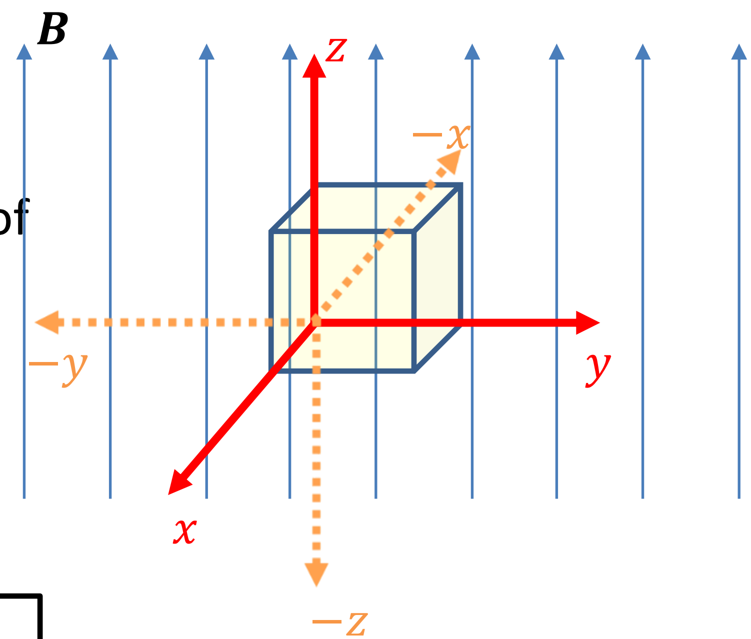
Magneto-Hydro-Dynamics

An Irreversible Thermodynamics Application

- Constitutive Equation between vectorial phenomena,

$$\boldsymbol{W} = \beta_1 \boldsymbol{V} + \beta_2 \tilde{B}(\boldsymbol{V}) + \beta_3 \tilde{B}[\tilde{B}(\boldsymbol{V})]$$

- where the coefficient β_k are scalar functions of the thermodynamic variables and $|B|^2$



$$\boldsymbol{W} = \beta_1 \boldsymbol{V} + \beta_2 \boldsymbol{V} \times \boldsymbol{B} + \beta_3 \boldsymbol{V} \times \boldsymbol{B} \times \boldsymbol{B}$$

«Isotropic» relation	«Hall» effect	«Across field-line» effect

Cartesian coordinate system with z-axis along magnetic field direction:

$$\begin{bmatrix} L_\perp & L_{cross} & 0 \\ -L_{cross} & L_\perp & 0 \\ 0 & 0 & L_{||} \end{bmatrix} = \begin{bmatrix} \beta_1 - \beta_3 B^2 & -\beta_2 B & 0 \\ \beta_2 B & \beta_1 - \beta_3 B^2 & 0 \\ 0 & 0 & \beta_1 \end{bmatrix}$$

Here $\boldsymbol{W}$ is either $\boldsymbol{i}^*$ or $\boldsymbol{K}_{\boldsymbol{q}}^*$ and $\boldsymbol{V}$ is either

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - \nabla_{\mathrm{T}} \mu_p^*$$

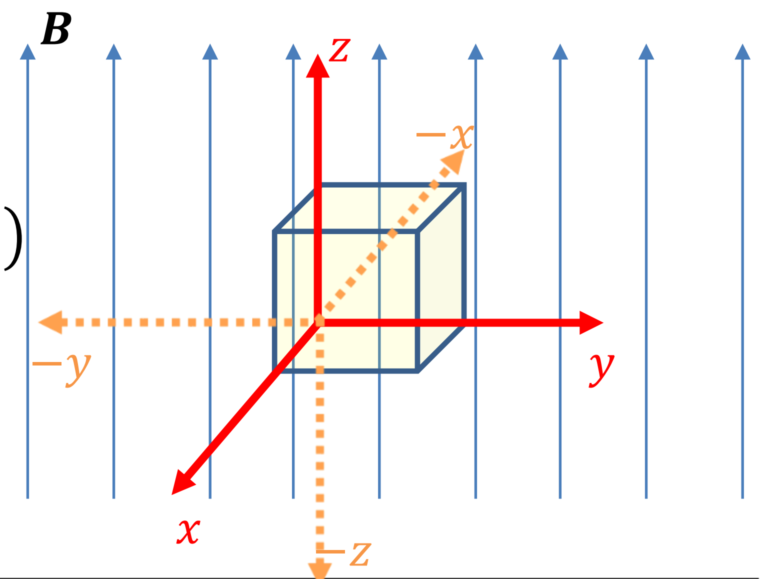
Or

$$-\nabla \mathrm{T}/\mathrm{T}$$

Magneto-Hydro-Dynamics

An Irreversible Thermodynamics Application

$$\underline{\Pi^{(s)}} = \eta_0 \dot{\nabla}\boldsymbol{v} + \eta_1\left(\dot{\nabla}\boldsymbol{v}\tilde{B} - \tilde{B}\dot{\nabla}\boldsymbol{v}\right) + \eta_2\left(\dot{\nabla}\boldsymbol{v}\tilde{B}^2 - \tilde{B}^2\dot{\nabla}\boldsymbol{v}\right) + \eta_3\left(\tilde{B}\dot{\nabla}\boldsymbol{v}\tilde{B}^2 - \tilde{B}^2\dot{\nabla}\boldsymbol{v}\tilde{B}\right) + \eta_4 \,\mathrm{tr}\left(\dot{\nabla}\boldsymbol{v}\tilde{B}^2\right)\left(\tilde{B}^2 - \mathrm{tr}\left(\tilde{B}^2\right)\underline{I}\right) + \eta_X\left[\nabla\cdot\boldsymbol{v}\left(\tilde{B}^2 - \mathrm{tr}\left(\tilde{B}^2\right)\underline{\boldsymbol{I}}\right) + \mathrm{tr}\left(\dot{\nabla}\boldsymbol{v}\tilde{B}^2\right)\underline{\boldsymbol{I}}\right] + \eta_v \nabla\cdot\boldsymbol{v}\,\underline{\boldsymbol{I}}$$



	$(\mathring{\nabla}\mathbf{v})^{(s)}_{xx}$	$(\mathring{\nabla}\mathbf{v})^{(s)}_{yy}$	$(\mathring{\nabla}\mathbf{v})^{(s)}_{zz}$	$(\mathring{\nabla}\mathbf{v})^{(s)}_{xy}$	$(\mathring{\nabla}\mathbf{v})^{(s)}_{yz}$	$(\mathring{\nabla}\mathbf{v})^{(s)}_{xz}$	$\nabla\cdot\mathbf{v}$
$\mathring{\Pi}_{xx}$	$\eta_0 - 2\eta_2 B^2 - \eta_4 B^4$	$-\eta_4 B^4$	0	$-2\eta_1 B - 2\eta_3 B^3$	0	0	ζB^2
$\mathring{\Pi}_{yy}$	$-\eta_4 B^4$	$\eta_0 - 2\eta_2 B^2 - \eta_4 B^4$	0	$2\eta_1 B + 2\eta_3 B^3$	0	0	ζB^2
$\mathring{\Pi}_{zz}$	$-2\eta_4 B^4$	$-2\eta_4 B^4$	η_0	0	0	0	$-2\zeta B^2$
$\mathring{\Pi}_{xy}$	$\eta_1 B + \eta_3 B^3$	$-\eta_1 B - \eta_3 B^3$	0	$\eta_0 - 2\eta_2 B^2$	0	0	0
$\mathring{\Pi}_{yz}$	0	0	0	0	$\eta_0 - 2\eta_2 B^2$	$\eta_1 B$	0
$\mathring{\Pi}_{xz}$	0	0	0	0	$-\eta_1 B$	$\eta_0 - 2\eta_2 B^2$	0
$\mathrm{tr}\,\Pi$	ζB^2	ζB^2	$-2\zeta B^2$	0	0	0	$-\eta_v$

Magneto-Hydro-Dynamics

An Irreversible Thermodynamics Application

- *Reaction rate*

$$J_r = k_r \rho_a \left[\exp\left(-\frac{m_a A_r}{k_B T} \right) - 1 \right]$$

For $J_r = 0$ and considering chemical affinity definition and the chemical potentials EoS

$$A_r = \sum_j \nu_j \mu_j$$

$$\mu_k = \frac{k_B T}{m_k} \log \bar{c_k} + \underbrace{\frac{k_B T}{m_k} \log \left[\frac{p}{k_B T} \frac{\Lambda_k^3}{G_k} \exp\left(\frac{\epsilon_{0,k}}{k_B T} \right) \right]}_{\zeta_k(p,T)}$$

Considering moreover $m_i/m_a \simeq 1$, $\mathrm{G_i}/\mathrm{G_a} \simeq 1$, $G_e = 2$, $\epsilon_i = \epsilon_{0,a} - \epsilon_{0,e} - \epsilon_{0,i}$

$$\frac{n_e n_i}{n_a} = \frac{(2\pi m_e k_B)^{3/2}}{h} T^{3/2} \exp - \frac{\epsilon_i}{k_B T}$$

which is the celebrated ***Saha Equation!***

2°

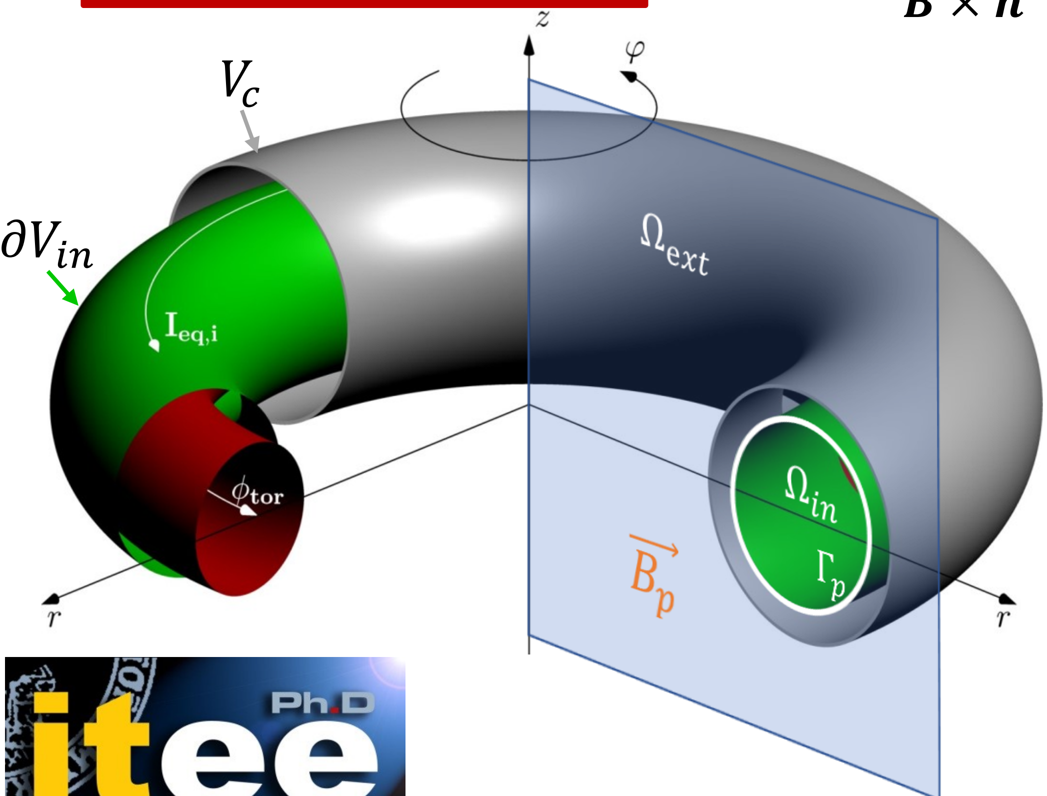
Coupling plasma MHD models with structures MQS models

MHD Plasma :
- *Differential formulation in V_{in}*

$\boldsymbol{A}^*_{pl}$ →

← $\boldsymbol{B} \times \hat{\boldsymbol{n}}$

MQS conductors:
- *Integral formulation in terms of potentials*



- Necessity of solving the outer MQS problem
- Possible *direct formulations* (Johnson-Nedelèc, in terms of $\boldsymbol{A}$ or $\boldsymbol{B}$)
- Implementing an *indirect* formulation based on the **Virtual Casing Principle**
  - The information in $\boldsymbol{A} \times \hat{\boldsymbol{n}}$ can be reproduced perfectly by an equivalent surface current
  - $\boldsymbol{k_{eq}}$ is div-free in absence of shared currents
  - $\boldsymbol{k_{eq}}$ can be generalized in presence of halo currents

Coupling plasma MHD models with structures MQS models

MHD Plasma :
- *Differential formulation in V_{in}*

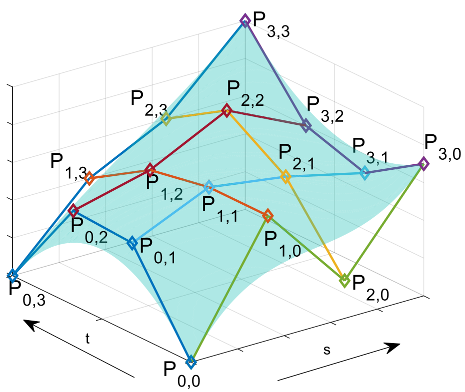
$\boldsymbol{A}_{pl}^{*}$ →

← $\boldsymbol{B} \times \hat{\boldsymbol{n}}$

MQS conductors:
- *Integral formulation in terms of potentials*

JOREK (e.g. *reduced MHD model)*

- $\boldsymbol{A} = \psi \nabla \varphi$
- Fourier in φ
- Bèzier patches in (r, z) plane

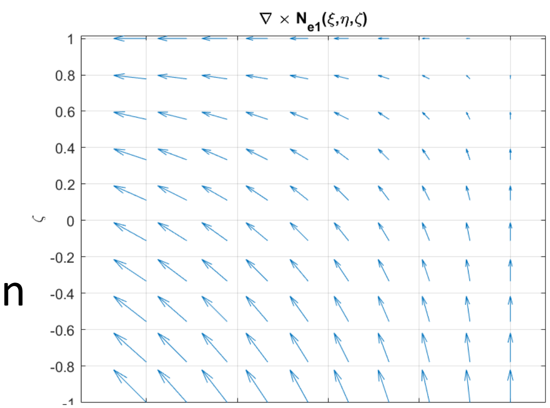


$$\frac{d}{dt}\underline{\psi} = \underline{\underline{P_{tan}}}\,\underline{B_{tan}} - \underline{\underline{M_{\psi}}}\,\underline{\psi} + [\dots]$$

$$B_{tan} = \frac{1}{r}\frac{\partial \psi}{\partial n}$$

CARIDDI

- $\boldsymbol{i} = \nabla \times \boldsymbol{T}$
- Edge basis vectors
- Tree-cotree decomposition



$$\underline{\underline{L_w}}\,\frac{\mathrm{d}}{\mathrm{d}t}\underline{I_w} + \underline{\underline{R_w}}\,\underline{I_w} + \underline{V_{pl}} + \underline{\underline{F_w}}\,\underline{V_{w,e}} = \underline{0}$$

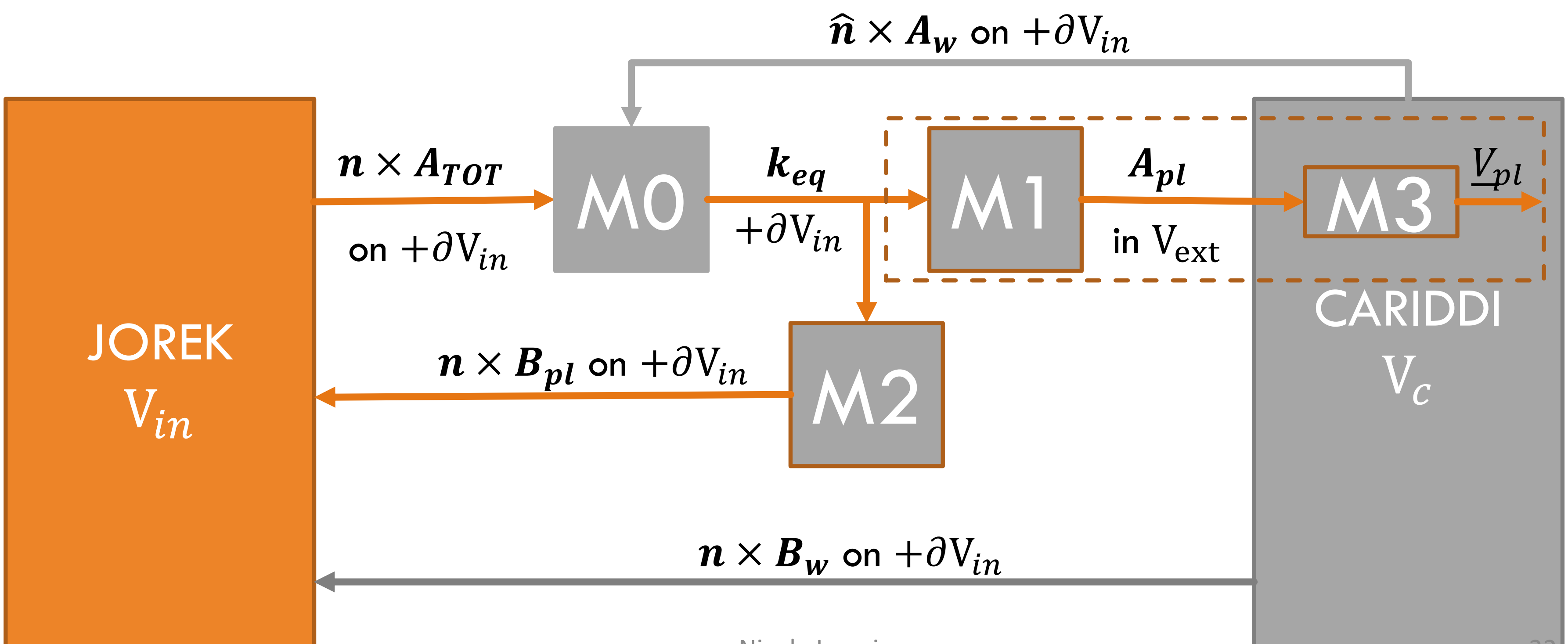
$$\underline{\underline{F}}^{T}\,\underline{I_w} = \underline{I_{w,e}}$$

$$V_{pl,k} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c} \mathbf{w_k} \cdot \mathbf{A}_{\mathrm{in}}^{*}\,\mathrm{d}\mathbf{r}'$$

$$I_{we,k} = \int_{S_k} \boldsymbol{i} \cdot \hat{\boldsymbol{n}}\,\mathrm{d}S$$

Coupling plasma MHD models with structures MQS models

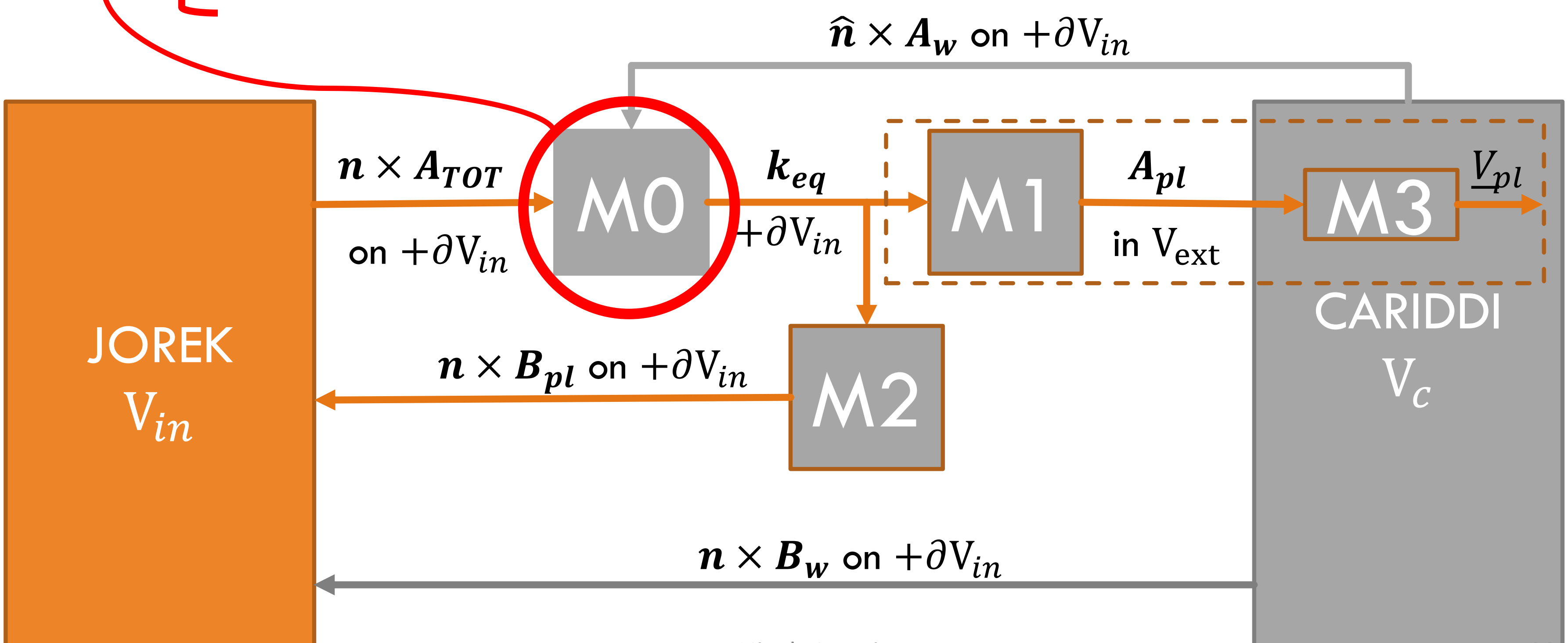
- Indirect boundary element method based on ***Virtual Casing Principle***



Coupling plasma MHD models with structures MQS models

$$\boldsymbol{A_{pl}}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int_{\partial\Omega_{pl'}} \frac{\boldsymbol{k_{eq}}(\boldsymbol{r'})}{|\boldsymbol{r} - \boldsymbol{r'}|} \mathrm{dS'}$$

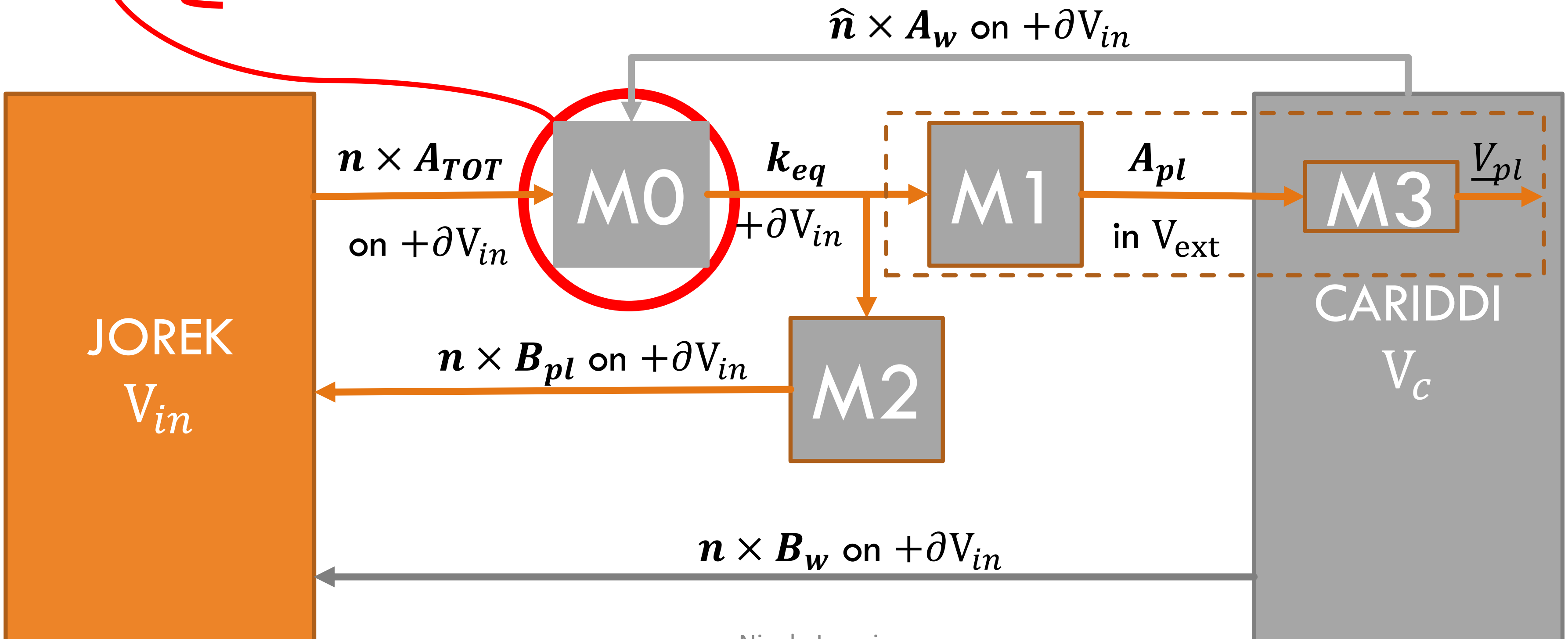
- The tangential component of the magnetic vector potential determines the «vacuum» field:



Coupling plasma MHD models with structures MQS models

- In weak form, for any $\boldsymbol{v} \in \ker[div(\partial\Omega_{pl})]$

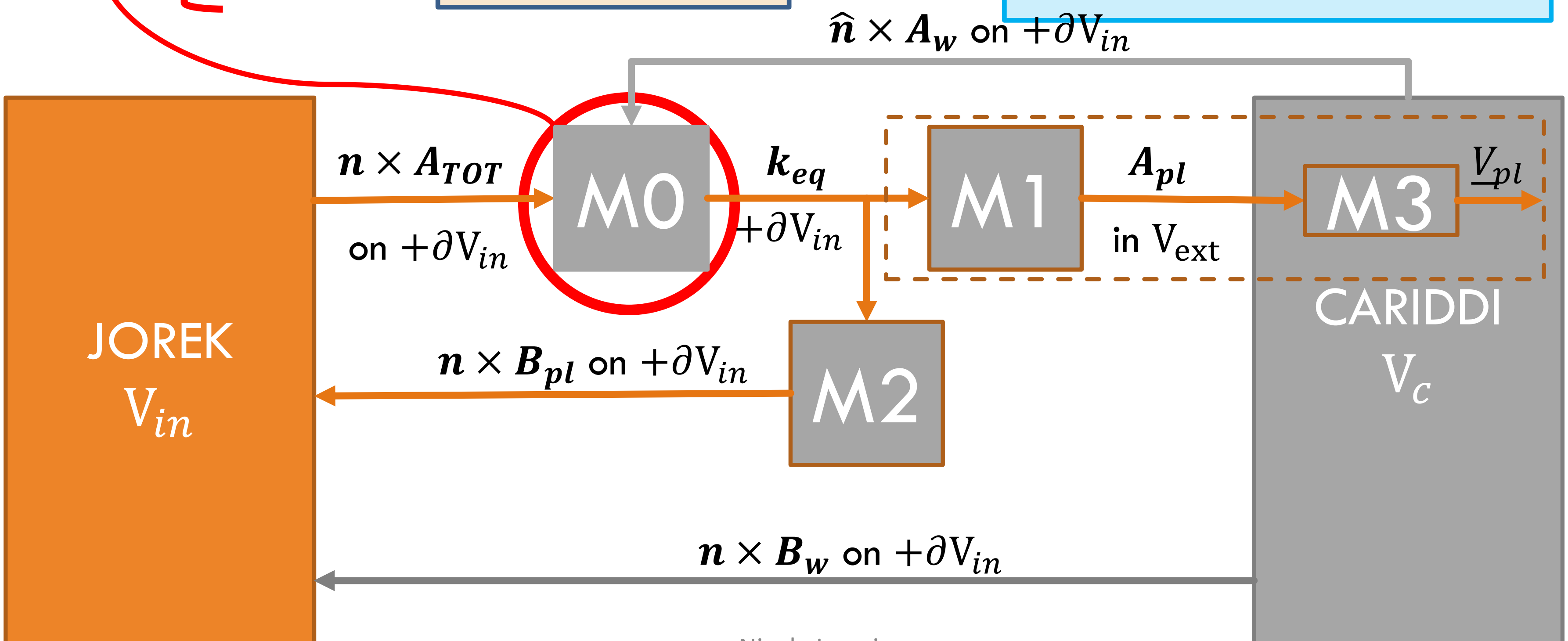
$$\int_{\partial\Omega_{pl}} \boldsymbol{A_{pl}}(\boldsymbol{r}) \cdot \boldsymbol{v}(\boldsymbol{r})\, \mathrm{d}S = \frac{\mu_0}{4\pi} \int_{\partial\Omega_{pl}} \int_{\partial\Omega_{pl'}} \frac{\boldsymbol{k_{eq}}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} \cdot \boldsymbol{v}(\boldsymbol{r})\, \mathrm{d}S'\, \mathrm{d}S$$



Coupling plasma MHD models with structures MQS models

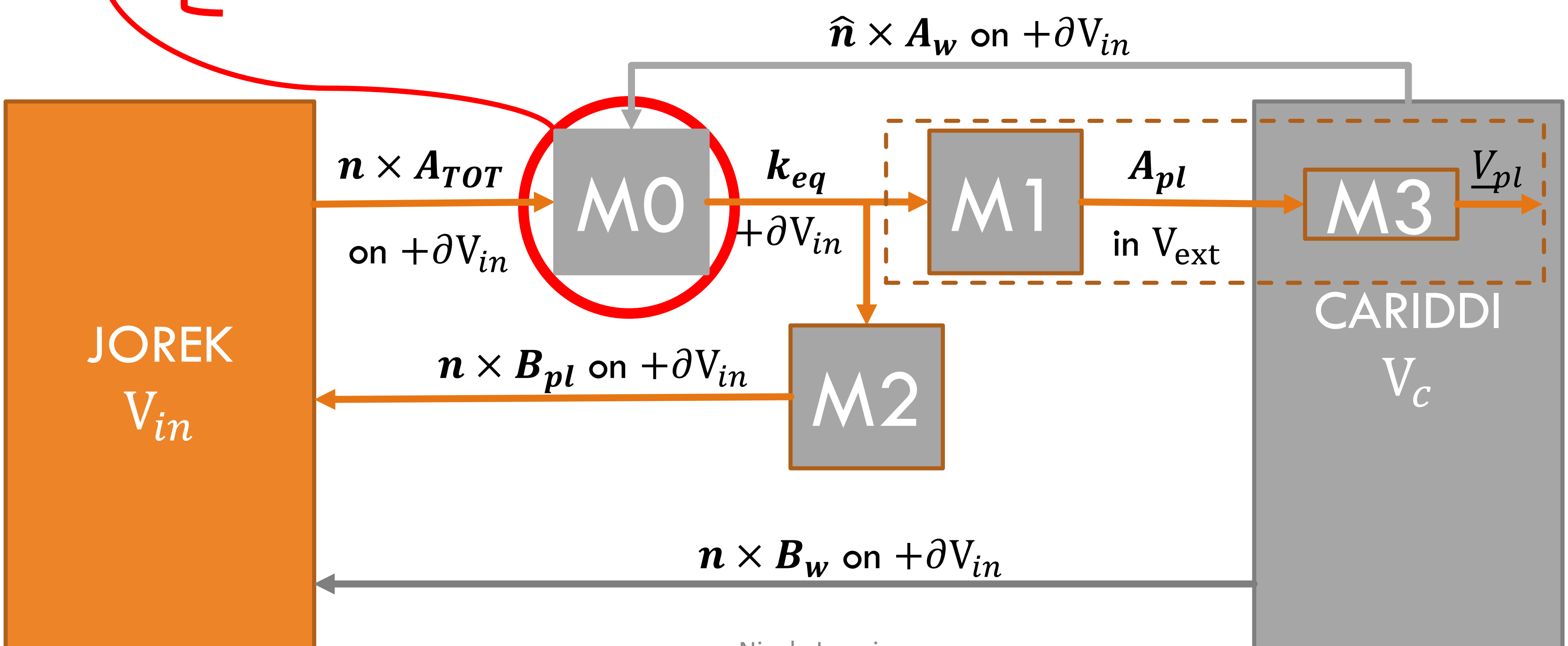
- CARIDDIshell at the JOREK boundary: $\{\mathbf{w_k} = \nabla \times \boldsymbol{T_k}\} \subset \ker[div(\partial\Omega_{pl})]$:

$$\sum_{j}^{N_{basis,J}} A_{pl,j} \underbrace{\int_V \boldsymbol{u_j}(\boldsymbol{r}) \cdot \boldsymbol{w_k}(\boldsymbol{r})\, \mathrm{d}V}_{H_{jk}} = \sum_{j}^{\mathrm{N_{edge}}} I_{eq,j} \underbrace{\frac{\mu_0}{4\pi} \int_V \int_{V'} \frac{\boldsymbol{w_j}(\boldsymbol{r'}) \cdot \boldsymbol{w_k}(\boldsymbol{r})}{|\boldsymbol{r} - \boldsymbol{r'}|}\, \mathrm{d}V'\, dV}_{(L_{eq})_{jk}}$$

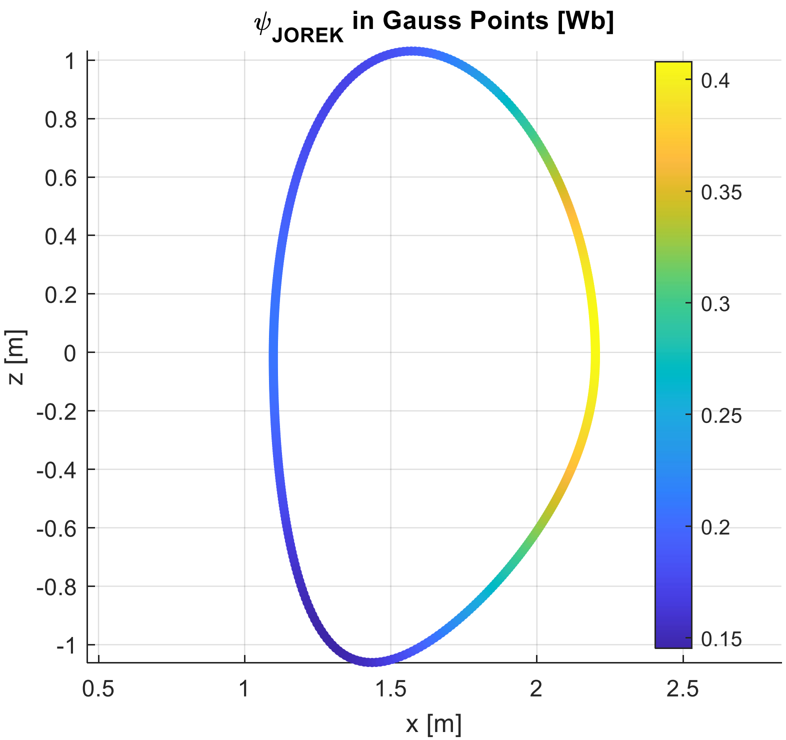


Coupling plasma MHD models with structures MQS models

- Creation of a thin volumetric shell overlapped to the JOREK boundary
- $\boldsymbol{A}_{pl} = \boldsymbol{A}_{TOT} - \boldsymbol{A}_{w}$
- $\underline{\boldsymbol{I}_{eq}} = \underline{\underline{\boldsymbol{L}_{eq}}}^{-1} \cdot \underline{\underline{\boldsymbol{H}}} \cdot \underline{\boldsymbol{\psi}_{pl}}$

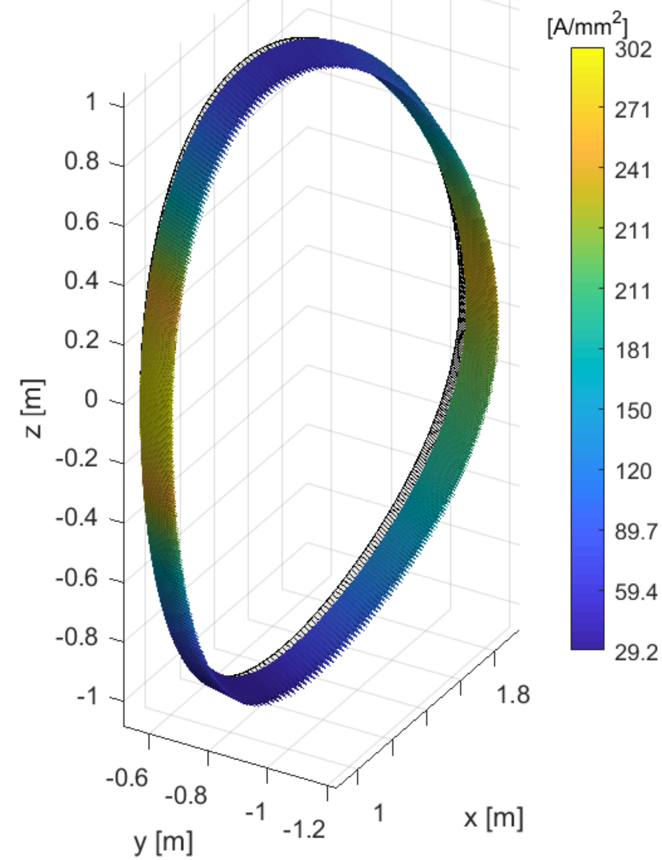


Coupling plasma MHD models with structures MQS models



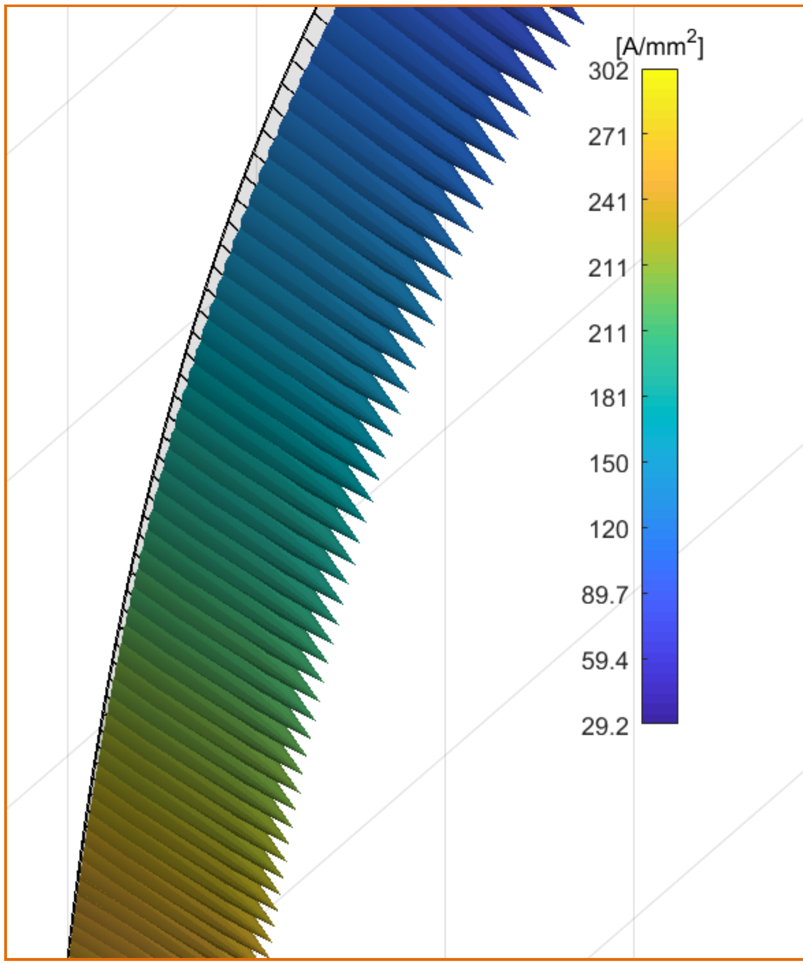
JOREK magnetic vector potential in CARIDDI Gauss points, represented by ψ.

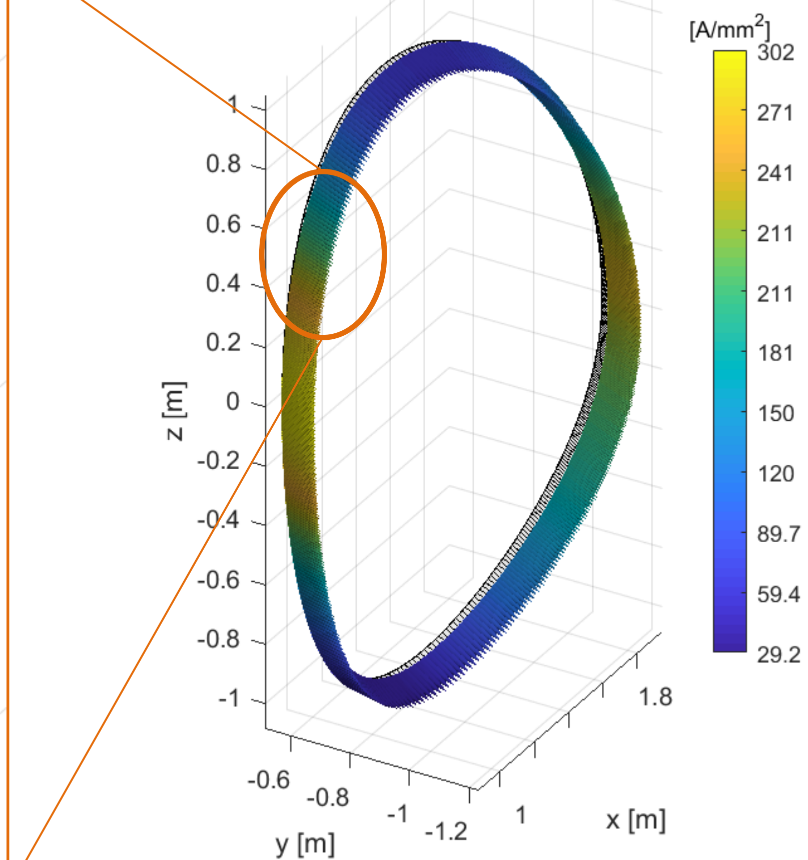
Corresponding Equivalent Surface Current.



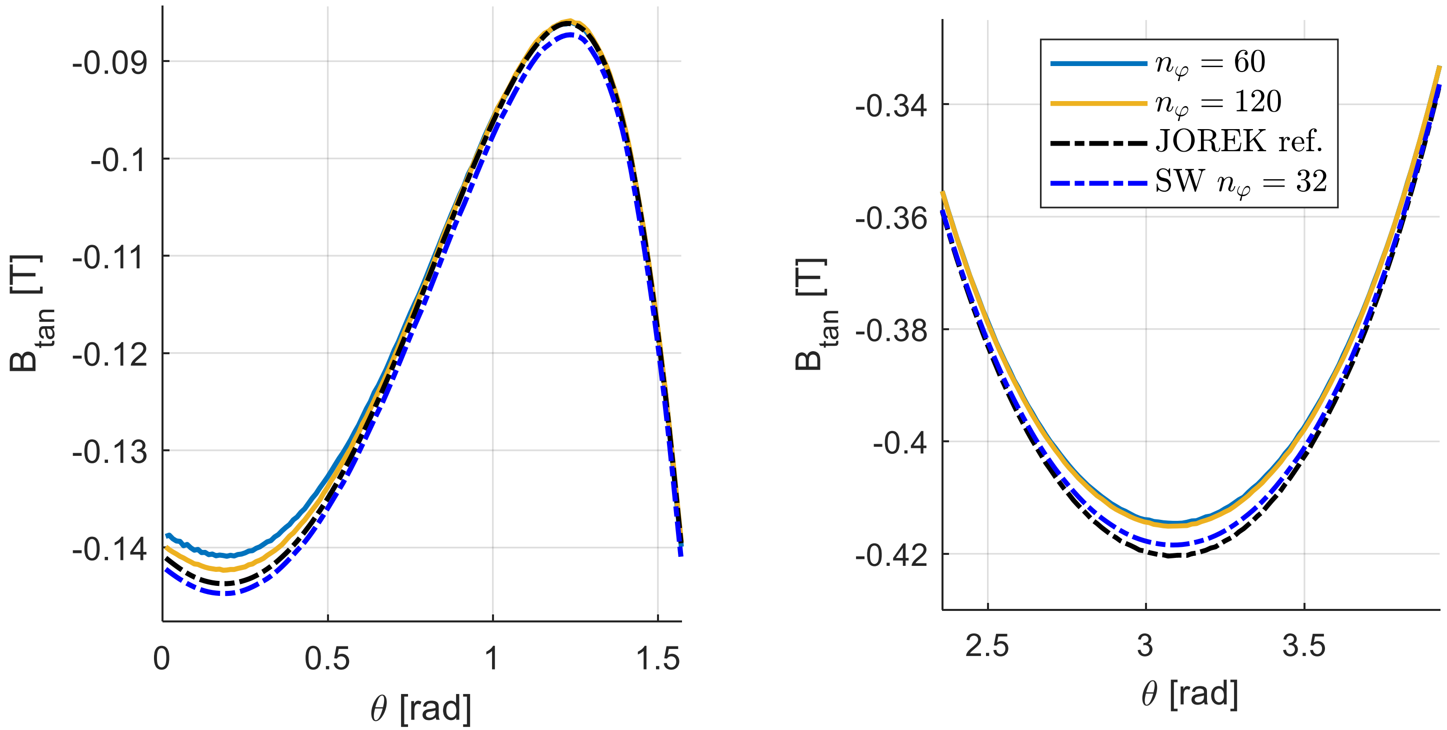
Coupling plasma MHD models with structures MQS models

Corresponding Equivalent Surface Current.





Coupling plasma MHD models with structures MQS models



Details of tangent magnetic field computed from the plasma-equivalent surface current, using different discretizations.

MHD *evolutionary* equilibrium

- *Eddy currents* offer a significant inertia to the plasma motion
- The plasma evolution is retained quasi-static

$$\boldsymbol{i} \times \boldsymbol{B} = \nabla p$$

- The current flows essentially within the Last Closed Flux Surface, and we formulate a **free-boundary problem**
- In toroidal geometry, with $\partial_\varphi = 0$, we get the **Grad-Shafranov Equation**

$$-r\nabla \cdot \left(\frac{\nabla\psi}{r^2}\right) = 2\pi r \frac{\mathrm{d}p}{\mathrm{d}\psi} + \frac{\mu_0}{8\pi^2 r}\frac{\mathrm{d}I^2}{\mathrm{d}\psi}$$

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Free-functions

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Free-functions

- Dirichlet problem with poloidal flux at the boundary obtained by the solution of outer problem
- The *free-boundary* formulation requires to specify the free functions in terms of a normalized flux

$$\tilde{\psi} = \frac{\psi - \psi_a}{\psi - \psi_b}$$

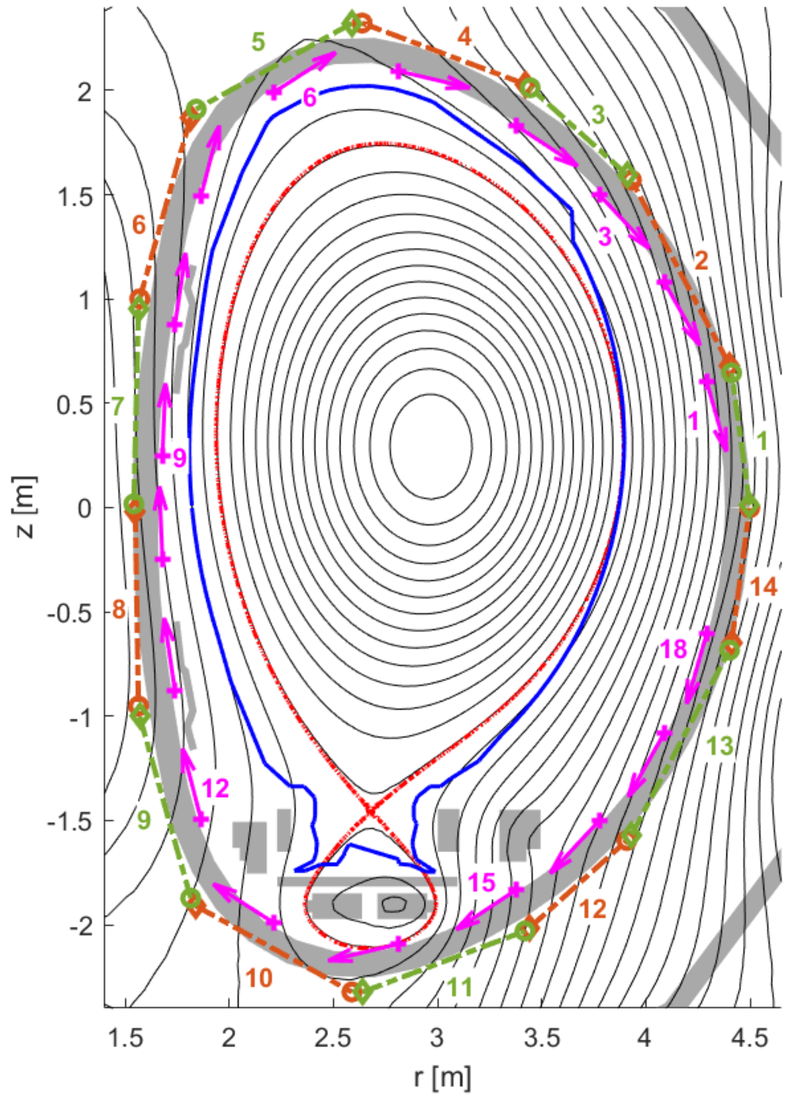
- Few parameters are sufficient for the current density distribution within the plasma, as only $\boldsymbol{B} \times \hat{\boldsymbol{n}}$ is of importance

$$\frac{\mathrm{d}p}{\mathrm{d}\psi} = \lambda \frac{\beta_0}{R_0}\left(1 - \tilde{\psi}^{\alpha_{m,p}}\right)^{\alpha_{n,p}}$$

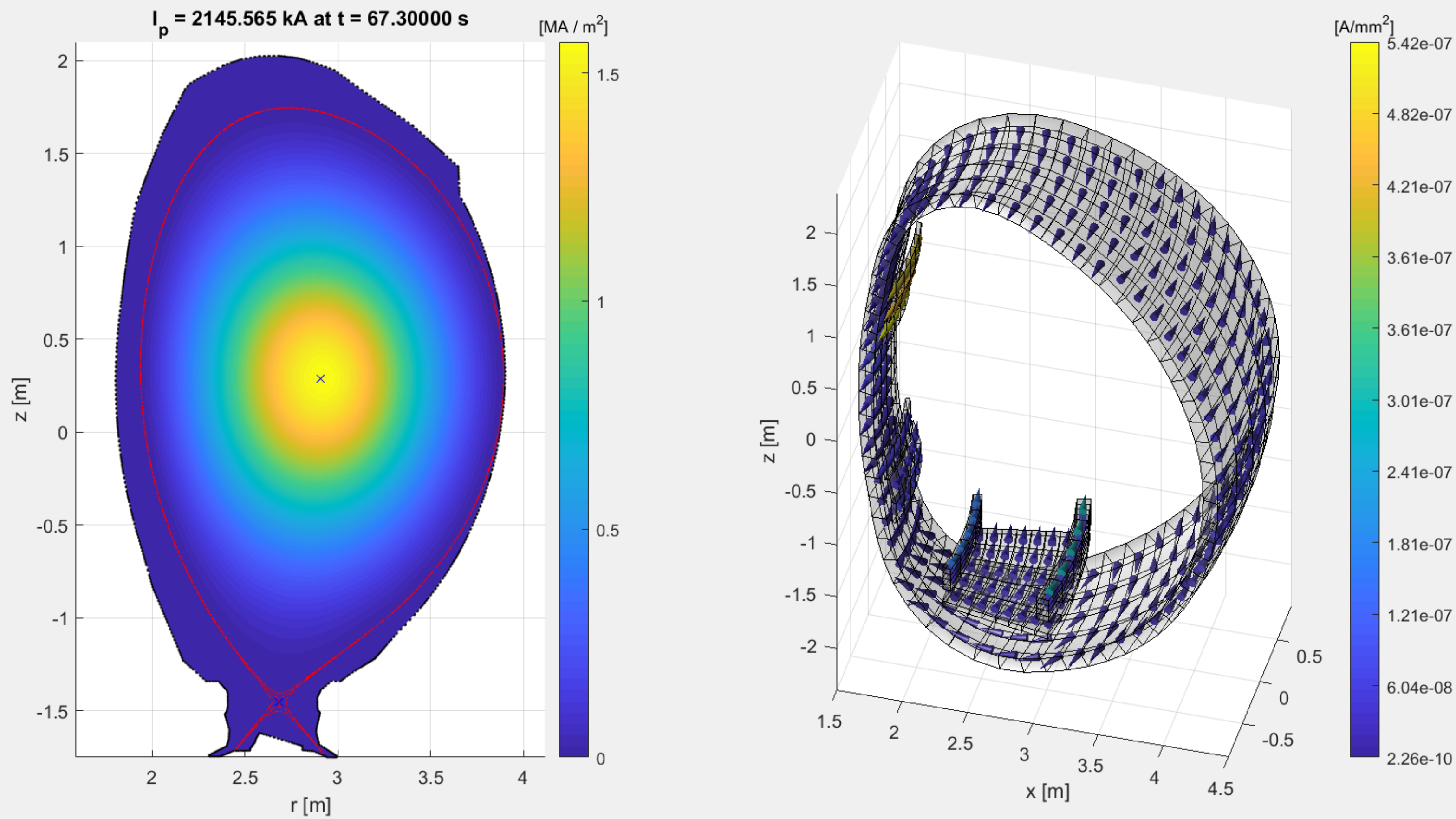
$$\frac{dI^2}{d\psi} = \frac{8\pi^2}{\mu_0}\lambda R_0(1 - \beta_0)\left(1 - \tilde{\psi}^{\alpha_{m,I}}\right)^{\alpha_{n,I}}$$

JET Vertical Displacement

- Iron core (deep saturation hypothesis)
- Largest tokamak operated ($R_w = 2.96\ m$)
- Large set of magnetic diagnostics
- Validation of MHD evolutionary equilibrium models
- Pulse #71985

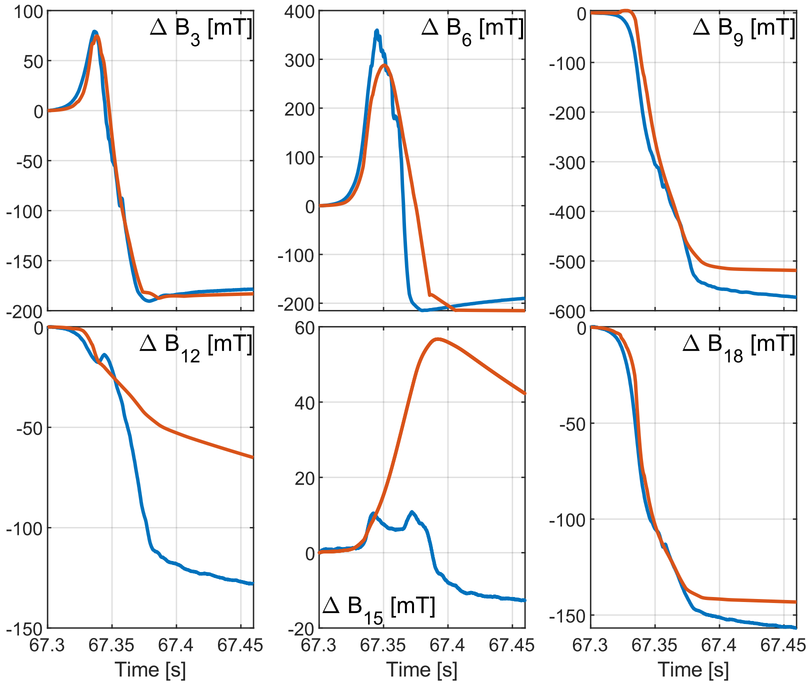


JET Vertical Displacement

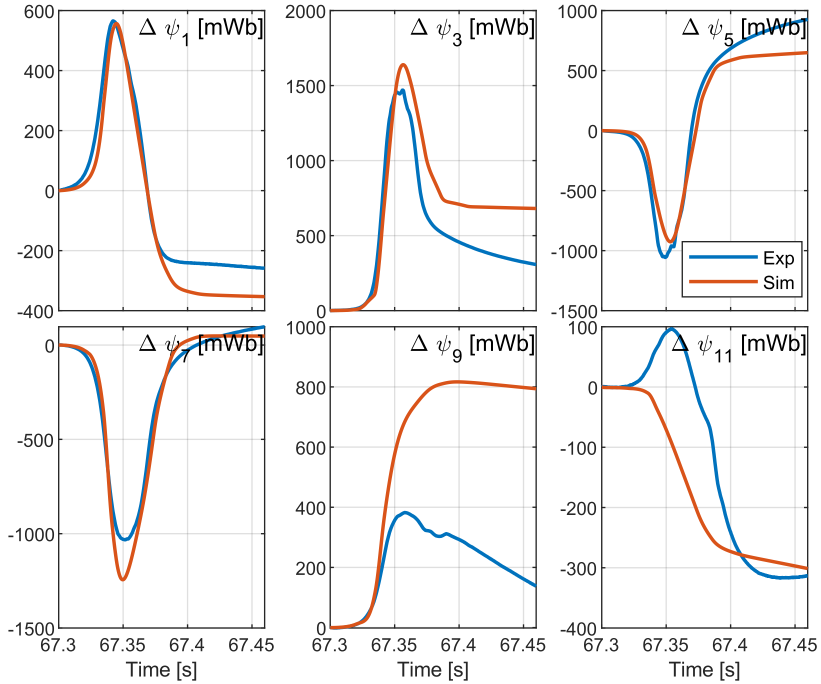


JET Vertical Displacement

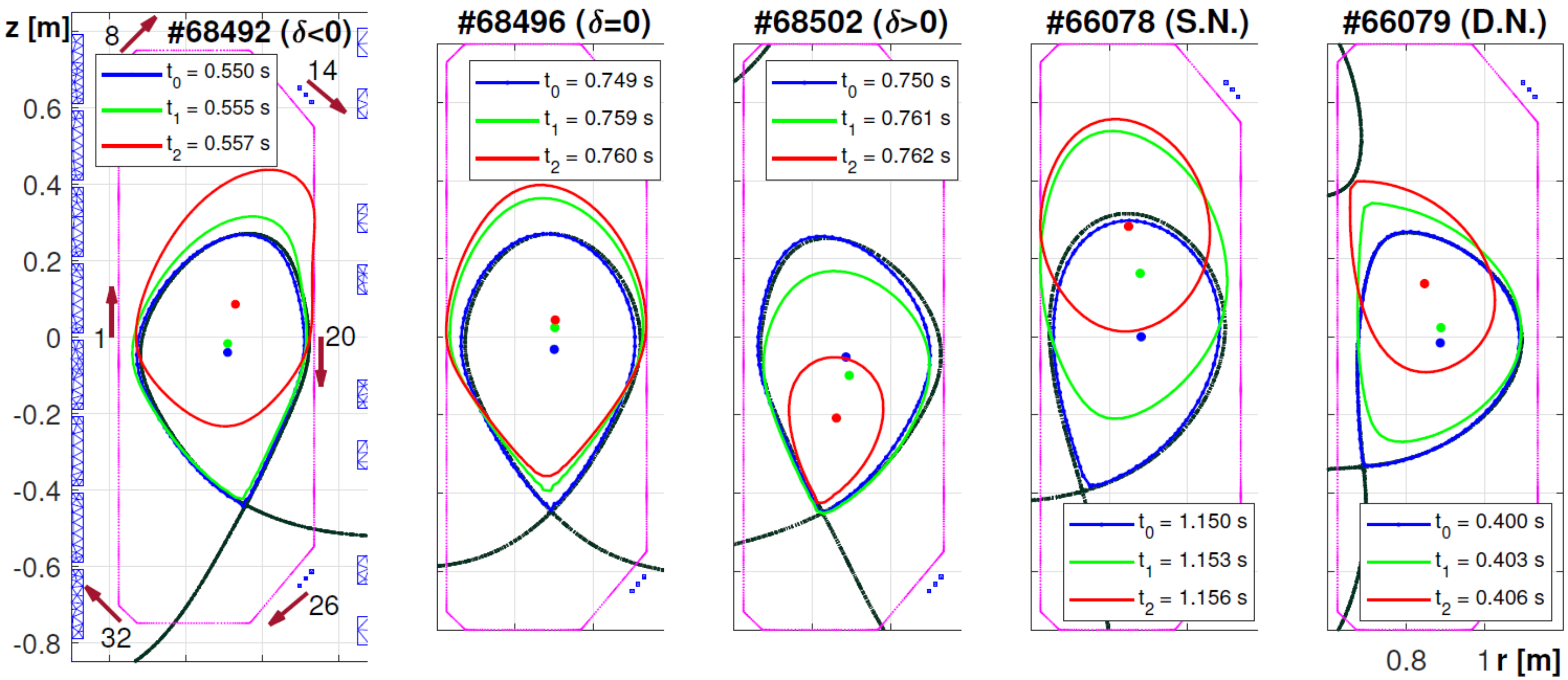
Internal Discrete Coils



Saddle Loops



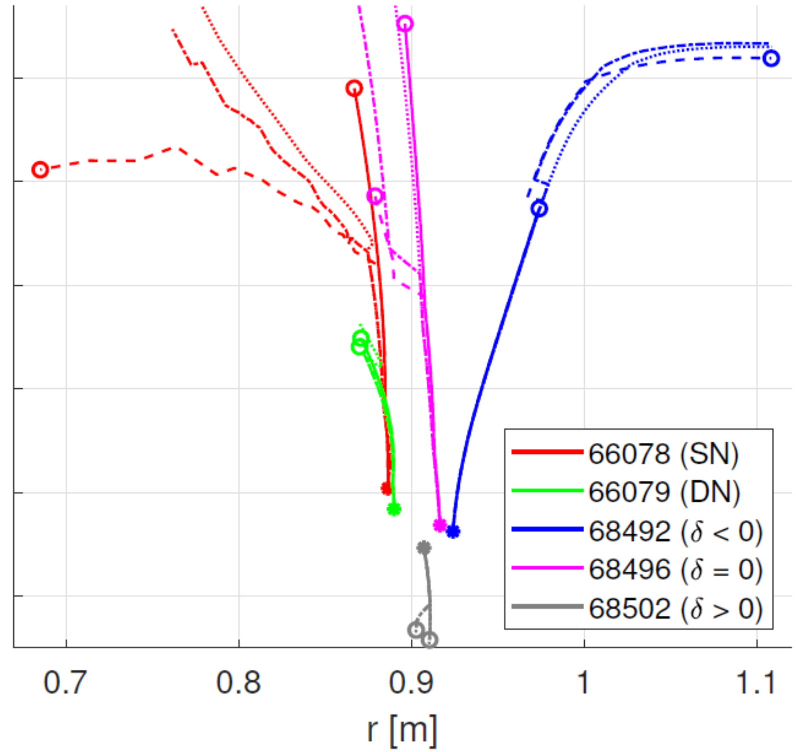
Disruption trajectory studies at TCV



- Necessity of predicting plasma-wall contact locations in DEMO
- Simulation campaign for experiments analysis

Disruption trajectory studies at TCV

- Positive triangularity plasmas tends to move inward
- Negative triangularity plasmas tends to move outward
- Plasma current decay amplifies above effects
- Thermal Quench displaces the plasma inboard
- The trajectory is not greatly affected by the growth rate



Solid lines: only voltage kick
Dashed: β_p drop, current overshoot and quench ($200\ kA/ms$)
Dotted: experimental plasma current
Dash-dotted: β_p drop and experimental current

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Disruption trajectory studies at TCV

- *High-aspect ratio* and *circular tokamak* ideal wall limit:

$$J(\Delta_b - \Delta_{iw}) = \text{const.}$$

Δ_b → $R_{pl} - R_w$

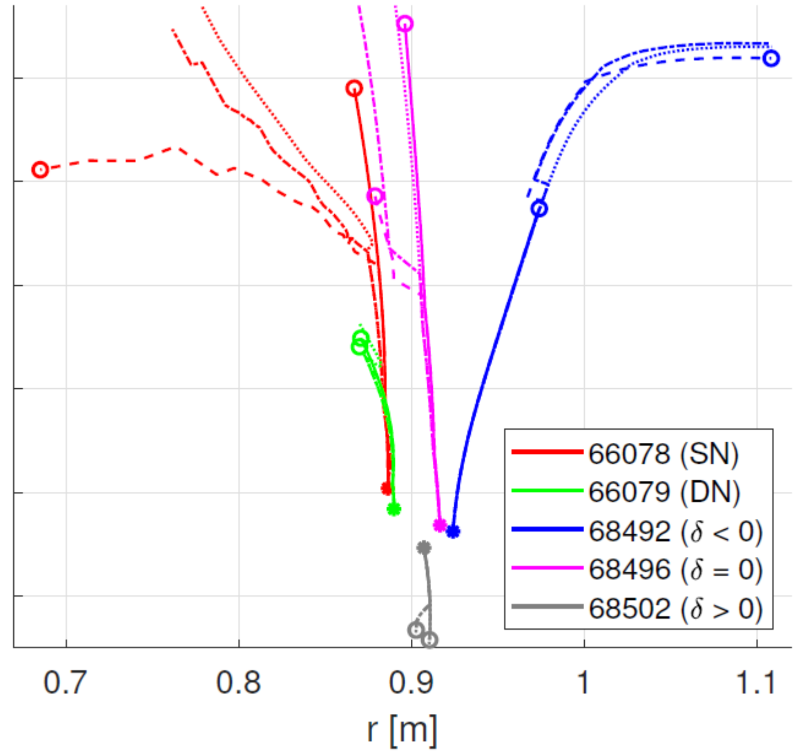
Δ_{iw} → $R_{iw} - R_w$

J → *net toroidal plasma current*

- Our proposal:

$\Delta_b > \Delta_{iw}$ for $\delta > 0$

$\Delta_b < \Delta_{iw}$ for $\delta < 0$



Solid lines: only voltage kick

Dashed: β_p drop, current overshoot and quench ($200\ kA/ms$)

Dotted: experimental plasma current

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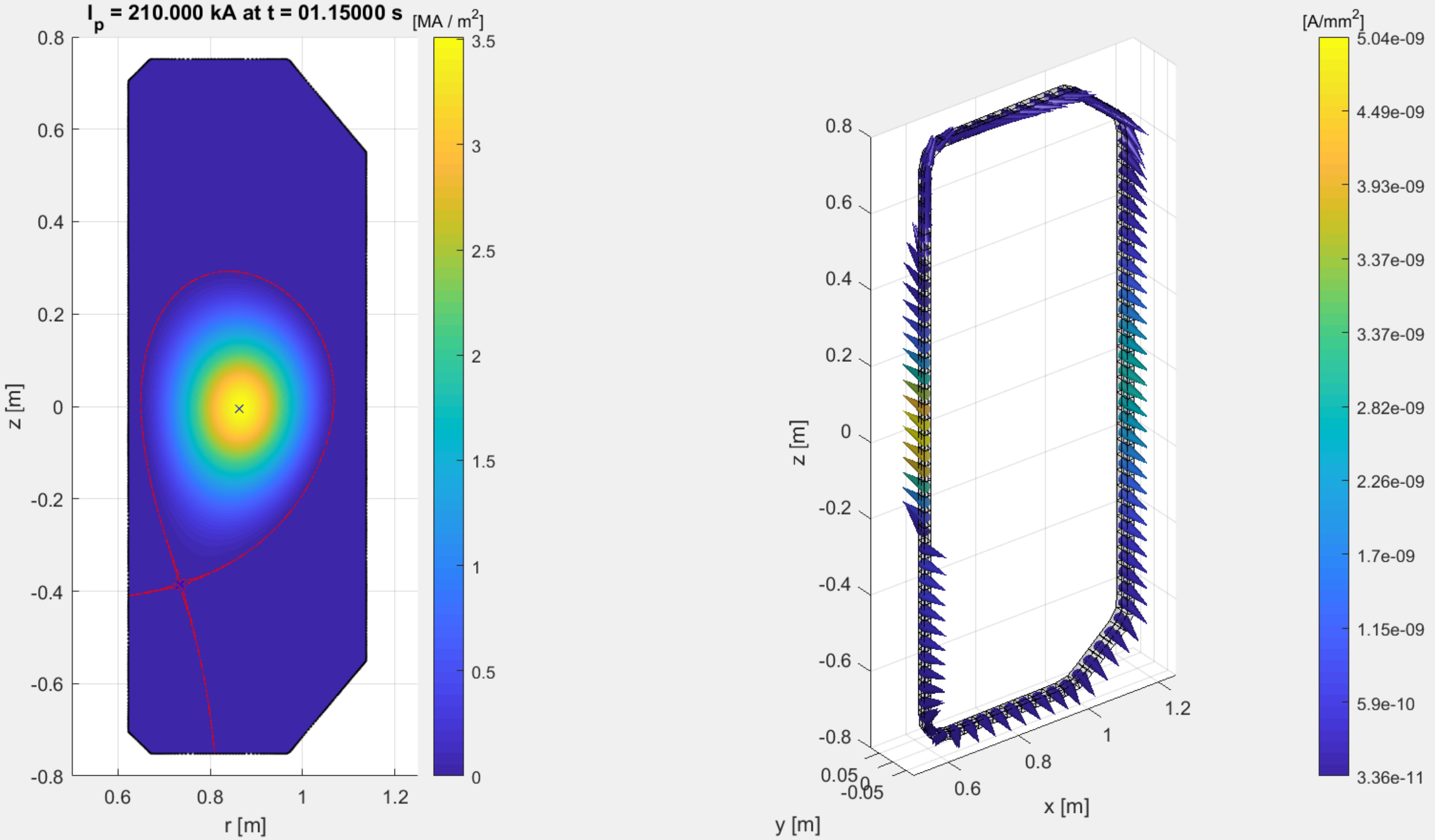
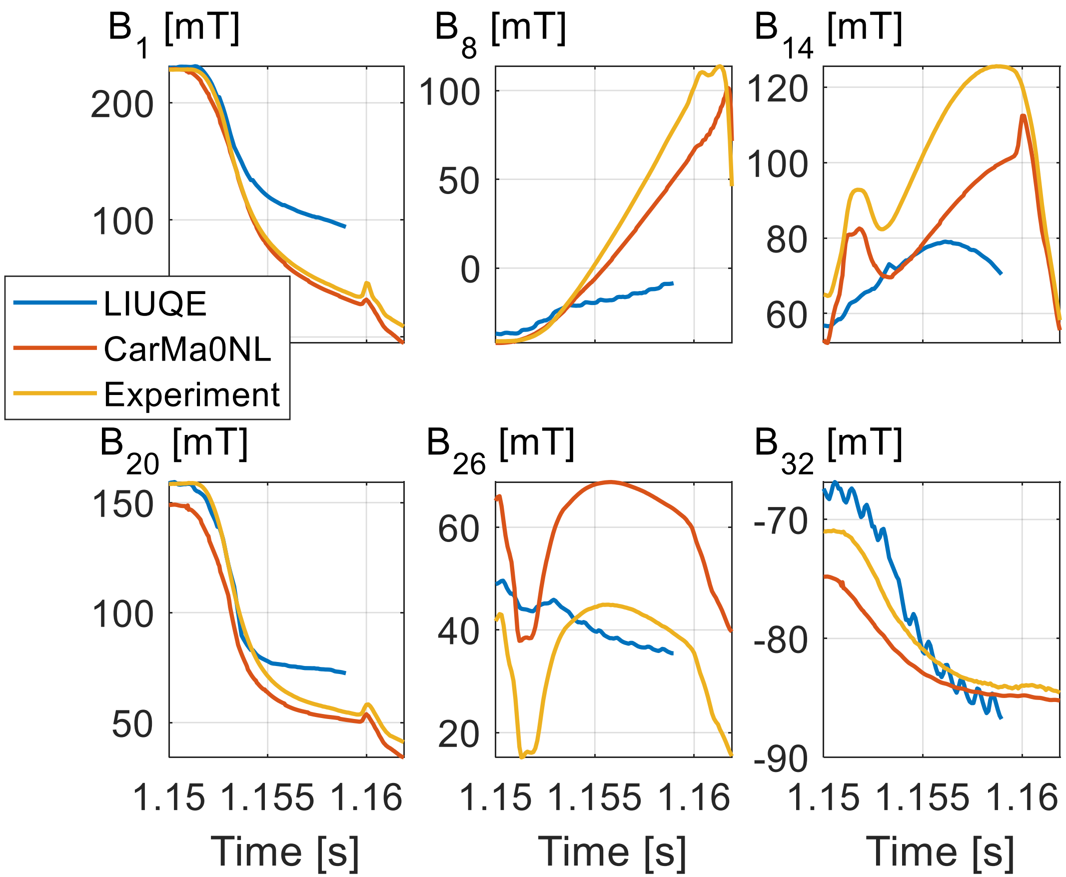
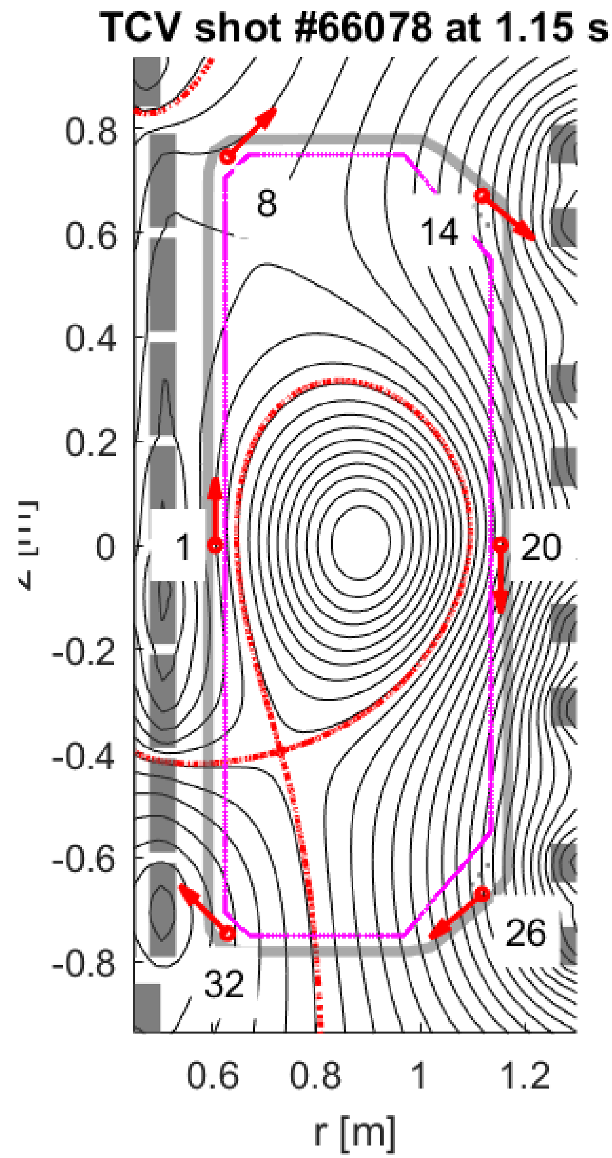
Disruption trajectory studies at TCV



Disruption trajectory studies at TCV





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SWISS PLASMA CENTER

Nicola Isernia

Resume

- It is possible to account for ionization reactions at the thermodynamic level
- The pseudo-vector nature of the magnetic field is important in studying the *symmetry* constraints on closure relations

- The coupling of non-linear extended MHD models with 3D volumetric MQS models is missing in the literature, besides important for *halo current* studies
- Implementation of an *indirect* method, based on the **Virtual Casing Principle**, for the JOREK-CARIDDI Coupling.
- Satisfactory preliminary results for the axisymmetric case

- Evolutionary equilibrium models describe correctly the electromagnetic evolution of some Tokamak experiments, validating the mass-less hypothesis
- Importance of the halo current in a realistic description
- The radial motion in off-normal events depends on the pre-disruption palsma shape and position

Thanks for your attention!

Questions?

Thanks for your attention

Questions?

Backup Slides

BACKUP SLIDES

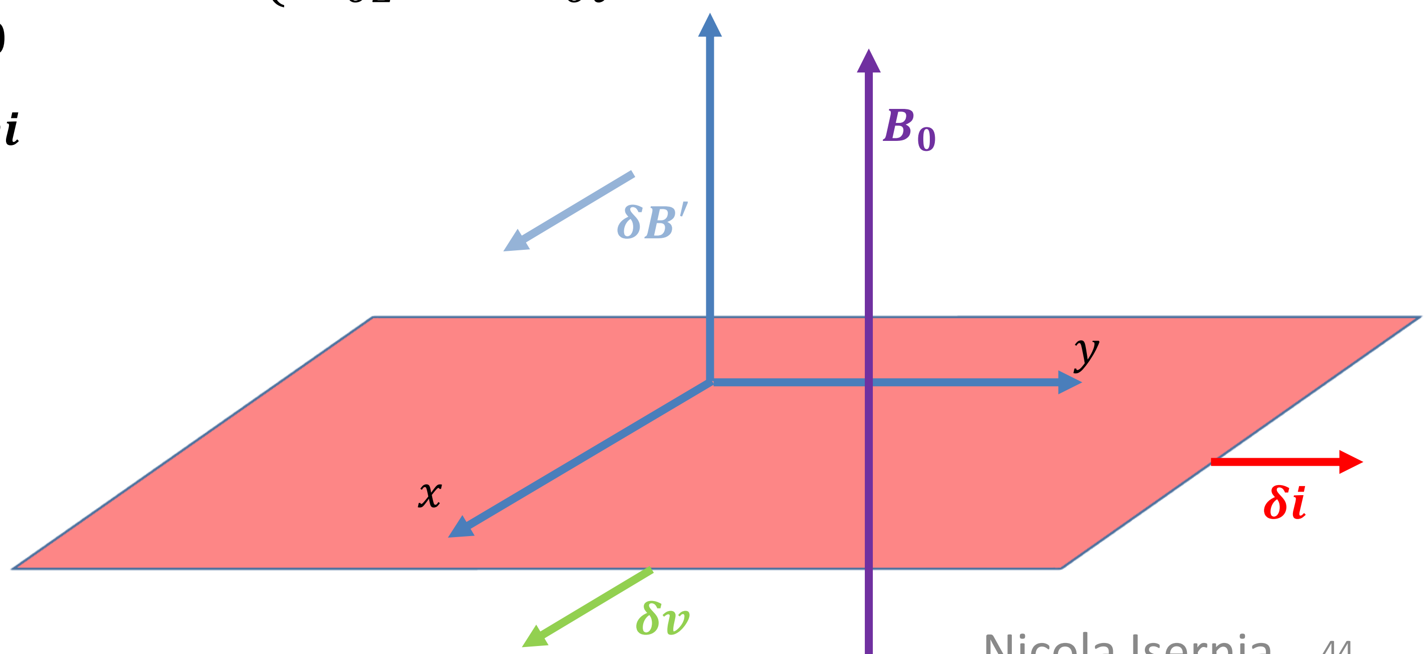
Fluid Conductors

- **1942** *H. Alfvèn* «Existence of electromagnetic-hydrodynamic waves», Nature, Vol. 150, 405-406

$$\begin{cases} \rho \dfrac{d}{dt}\delta \boldsymbol{v} = \mathbf{i} \times \boldsymbol{B} \\ \nabla \times \boldsymbol{B} = \mu_0 \mathbf{i} \\ \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = -\dfrac{\partial}{\partial t}\boldsymbol{B} \end{cases} \rightarrow \begin{cases} \rho_0 \dfrac{d}{dt}\delta v = B_0 \delta i \\ -\dfrac{\partial}{\partial z}\delta B = \mu_0 \delta i \\ B_0 \dfrac{\partial}{\partial z}\delta v = -\dfrac{\partial}{\partial t} B \end{cases} \rightarrow \left(\frac{B_0}{\sqrt{\mu_0 \rho_0}}\right)^2 \frac{\partial^2}{\partial z^2}\delta B - \frac{\partial^2}{\partial t^2}\delta B = 0$$

$$\uparrow \ \eta \simeq 0$$

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \eta \boldsymbol{i}$$



Magneto-Hydro-Dynamics

An Irreversible Thermodynamics Application

- An ideal gas mixture in local equilibrium

$$s_k = s_k(c_k, u_k)$$

- Sackur-Tetrode formula

$$s_k = \frac{k_B}{m_k}\left(\log\left[\frac{1}{n_k\Lambda_k^3}\right] + \frac{5}{2} + \log G_k\right)$$

- Internal energies

$$u_k = \frac{3}{2}\frac{k_B T_k}{m_k} + \frac{\epsilon_{0,k}}{m_k}$$

- Overall specific entropy

$$s = \sum_k c_k\, s_k$$

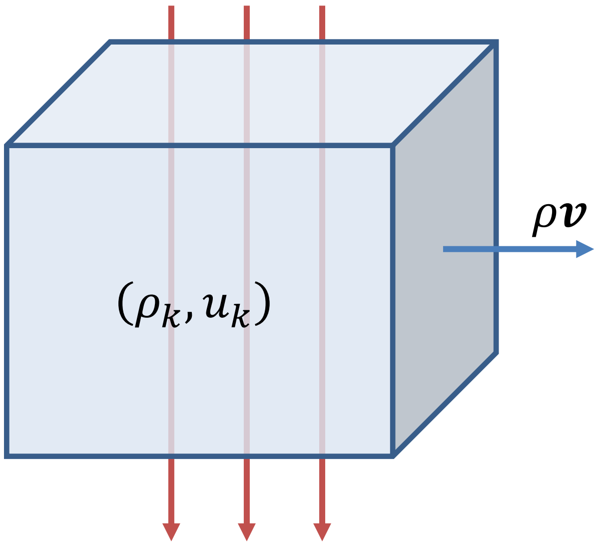
- Temperature equilibration and internal energy definition

$$T_e = T_i = T_a = T \;\rightarrow\; u = \sum_k c_k\, u_k$$

- First principle of thermodynamics for a reversible process

$$\mathrm{d}u = T\mathrm{d}s - p\mathrm{d}v + \sum_k \mu_k \mathrm{d}c_k$$

$$c_k = \frac{\rho_k}{\rho}$$



$$\boldsymbol{j_k} = \rho_k(\boldsymbol{v_k} - \boldsymbol{v})$$

- $\Lambda_k = \frac{h}{\sqrt{2\pi m_k k_B T_k}}$ Thermal De Broglie wave-length
- G_k degeneracy of ground state
- $\epsilon_{0,k}$ intrinsic energy of ground state

Magneto-Hydro-Dynamics

An Irreversible Thermodynamics Application

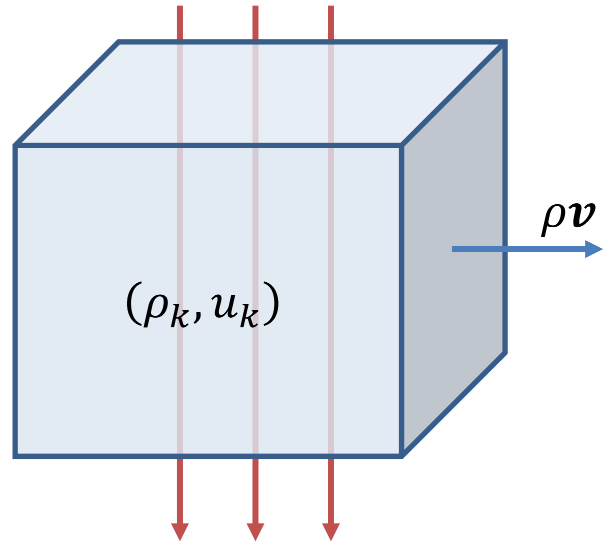
- $c_k = \frac{\rho_k}{\rho}, \;\; n_k = \frac{\rho_k}{m_k}, \;\; \bar{c_k} = \frac{n_k}{n}, \;\; \rho v = \sum_k \rho_k \boldsymbol{v_k}$
- Equations of State (Thermal Equilibrium)

$$\rho u = \frac{3}{2}p, \quad p = \sum_k \overbrace{n_k k_B T}^{p_k}$$

$$\mu_k = \frac{k_B T}{m_k} \log \bar{c_k} + \underbrace{\frac{k_B T}{m_k} \log\left[\frac{p}{k_B T}\frac{\Lambda_k^3}{G_k}\exp\left(\frac{\epsilon_{0,k}}{k_B T}\right)\right]}_{\zeta_k(p,T)}$$

- Conservation laws

$$\begin{cases} \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \boldsymbol{v} = 0 \\ \frac{\partial q}{\partial t} + \nabla \cdot \boldsymbol{i} = 0 \\ \rho_a \frac{\mathrm{d}}{\mathrm{d}t} c_a + \nabla \cdot \boldsymbol{j_a} = \nu_a J_r \\ \rho \frac{d}{dt}\boldsymbol{v} + \nabla p + \nabla \cdot \underline{\boldsymbol{\Pi}} = q\boldsymbol{E} + \boldsymbol{i} \times \boldsymbol{B} \\ \frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \boldsymbol{v} + \boldsymbol{K_q}) = -p \nabla \cdot \mathbf{v} - \underline{\boldsymbol{\Pi}} : \nabla \mathbf{v} + \mathbf{i}^* \cdot (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \end{cases}$$

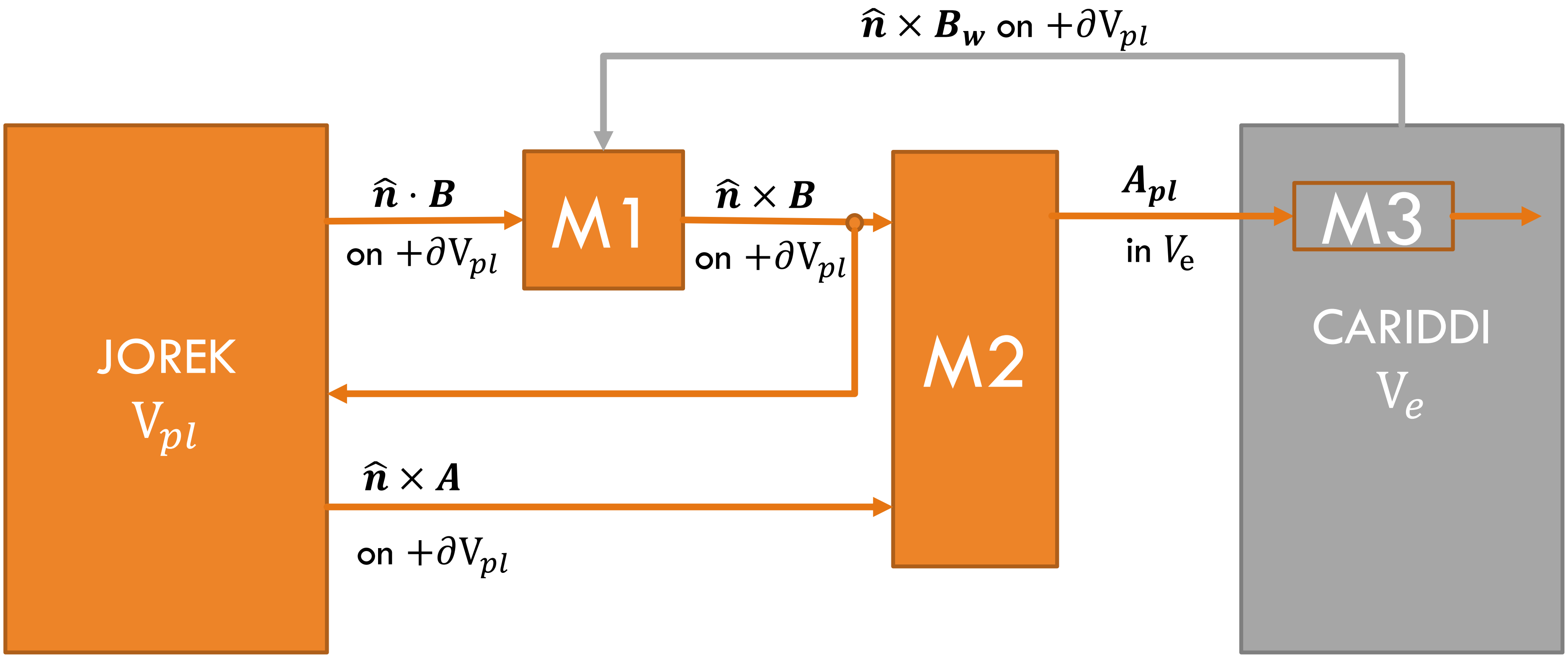


$$\boldsymbol{j_k} = \rho_k(\boldsymbol{v_k} - \boldsymbol{v})$$

- $\boldsymbol{i}^* = \sum_k \frac{\boldsymbol{e_k}}{\boldsymbol{m_k}} \boldsymbol{j_k}$
- $\boldsymbol{i} = \sum_k \frac{\boldsymbol{e_k}}{\boldsymbol{m_k}} \boldsymbol{\rho_k v_k}$
- $\boldsymbol{i}^* = \boldsymbol{i} - q\boldsymbol{v}$

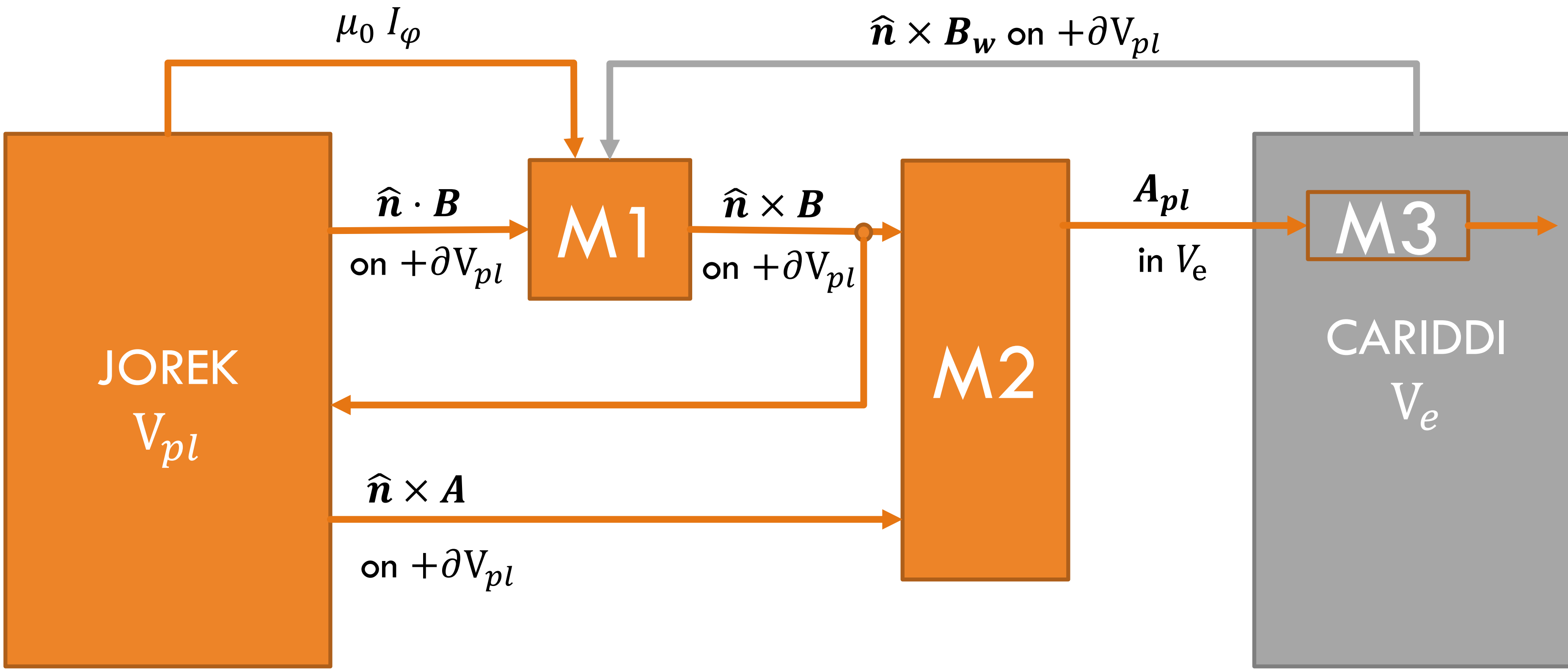
Direct-B Method

- For $n > 0$



Direct-B Method

- For $n = 0$



Direct-A Method

- For $n = 0$
- Pathological for $n \neq 0$ in Gauges different from Coulomb