Giovanni Gravina
Tutor: Carlo Forestiere
XXXIV Cycle - III year presentation

A basis of static surface modes for the scattering from penetrable objects

Università degli Studi di Napoli Federico II

My background

Graduation:

- B.Sc. degree in Electronic Engineering at the University of Naples “Federico II” on October 5, 2016.
  Thesis: “DATARIV: Profilazione utenti e sicurezza dell’applicazione”
- M.Sc. degree cum laude in Electronic Engineering at the University of Naples “Federico II” on October 25, 2018.
  Thesis: “Sulle risonanze di superfici conduttive”

Fellowship:

- PhD Student of XXXIV cycle in Information Technology and Electrical Engineering (ITEE).
  Theme: “Development of new Spectral Methods”

PhD Context

- Research Group: Prof. Giovanni Miano, Prof. Carlo Forestiere (tutor), Prof. Gugliemo Rubinacci, PhD student Bruno Miranda

- I accomplished the PhD program without a grant

- As an officer of the Italian Air Force, I am currently employed at 10^{th} Aircraft Maintenance Depot with the following responsabilities
  - to ensure the airworthiness of the Italian training fleet
  - to verify both on the ground and in flight the successful fulfilment of maintenance operations

Credit summary

	Credit Year 1						Credit Year 2						Credit Year 3							Summary	Check
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	7		
	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Bim	Tri		
Modules	0	3	0	0	6	6	0	0	4	0	0	0	9	9	10	0	4	0	0	**51**	**30-70**
Seminars	0	2	0	0	0	2	0	0	2	5	0	3	0	0	0	0	0	0	0	**14**	**10-30**
Research	3	3	3	4	4	4	8	8	7	7	8	8	7	7	7	7	7	7	6	**115**	**80-140**

Modules:

Year	Module Title	Type	Credits	Lecturer
3	Nanotechnology for Electrical Engineering	Ms Module	9	Prof. Forestiere
3	Introduzione ai Circuiti Quantistici	Ms Module	9	Prof. Miano
3	Avionics	External Module	2	Cap. Silvana Mele
3	Performance	External module	2	Maj Alessandro D'Argenio
3	Propulsion	External Module	2	Maj. Jacopo Covioli
3	Flying And Handling qualities & FCS	External Module	2	Cap. Filippo RInaldi
3	Armament	External Module	2	Maj. Raffaele Quartucci
3	Flight Test	External Module	4	Maj. Emiliano Battistelli

List of pubblications

Journal papers:

- C. Forestiere, G. Gravina, G. Miano, M. Pascale, R. Tricarico “Electromagnetic modes and resonances pf two-dimensional bodies”, Physical Review B. B 99, 155423 (2019)

- C. Forestiere, G. Gravina et al. “Static surface mode expansion for the full-wave scattering from penetrable objects”, IEEE Trans. Antenna Prapag. (submitted), arXiv: 2201.11058 (2022)

Agenda

- Context
- Motivation
- Static Basis
  - Static Longitudinal current modes
  - Static Transverse current modes
  - Numerical generation of the static basis
  - Orthogonality and Gram matrices
- Poggio-Miller-Chang-Harrington-Wu-Tsai Surface Integral Formulation
- Galerkin projection
- Result Analysis
  - Sphere
  - Rod
- Conclusions

Context

The resonant electromagnetic behavior of nanostructures is essential for the analysis and engineering of the field-matter interaction

Compared to the direct solution, the characterization of the scattering by nanostructure in terms of resonances and modes

- offers intuitive insights into the physics of the problem
- enables the rigorous comprehension of interference phenomena, including Fano resonances, as the interplay among well-identified modes
- suggests how to shape the excitation to achieve a prescribed tailoring of the scattering response

$$-Ax = \lambda x$$

$$x = \sum_i c_i x_i = \sum_i \frac{1}{\beta - \lambda_i} \frac{\langle x_i | b \rangle}{\langle x_i | x_i \rangle} x_i$$

Motivation

Accurate and efficient solutions of integral formulations heavily depends on the choice of basis functions.

- Analytic entire domain basis function may be generated in all coordinate system where the Helmholtz equation is separable
- A different strategy to generate entire domain basis function even in irregular domains is by introducing a convenient auxilirty eigenvalue problem (Example: Characteristic modes)

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Subject of the research is to look for a basis set that can simultaneously simplify the numerical solution of **electromagnetic scattering problems from a given object at multiple frequencies**

Static Basis

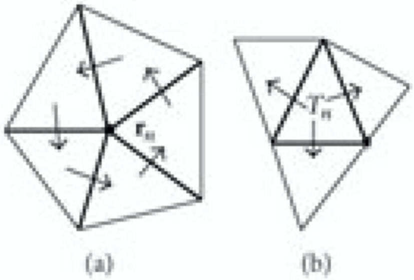
Key Idea: Helmholtz decomposition → any sufficiently smooth vector field can be resolved into the sum of two terms

- An irrotational and non-solenoidal vector field
- A solenoidal and rotational (non zero curl) vector field

Static longitudinal current modes

Static transverse current modes

Convenient low level (sub-domain) basis functions: **loop-star**



Static longitudinal current modes

The longitudinal modes are solution of the auxiliary eigenvalue problem

$$\mathcal{T}_0^{\parallel}\left\{\mathbf{j}_k^{\parallel}\right\}(\mathbf{r}) = \gamma_k^{\parallel}\mathbf{j}_k^{\parallel} \quad \text{where} \quad \mathcal{T}_0^{\parallel}\{\mathbf{w}\}(\mathbf{r}) = \hat{\mathbf{n}} \times \hat{\mathbf{n}} \times \nabla \oint_{\partial\Omega} g_0(\mathbf{r}-\mathbf{r}') \nabla'_S \cdot \mathbf{w}(\mathbf{r}')\, dS',$$

Static Green function

Its spectrum has the following properties

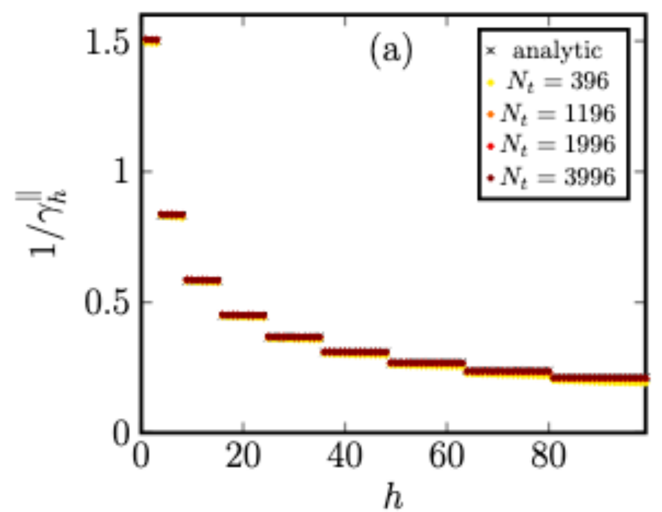
- The set of eigenvalues and the set of eigenfunctions are infinite countable
- The eigenvalues are real and positive
- The eigenfunctions associated to non-degenerate eigenvalues are orthogonal according to the scalar product

Static longitudinal current modes

- The eigenvalues and the modes depend on the shape of the particle, but are independent of both particle material and frequency of operation
- For a spherical surface of unit radius the eigenvalues have the following analytical expression

$$\frac{1}{\gamma_k^{\parallel}} = \frac{(2n+1)}{n\,(n+1)}.$$

The eigenvalues are compared with their analytical counterpart



Static transverse current modes

The transverse modes are solution of the auxiliary eigenvalue problem

$$\mathcal{T}_0^{\perp}\left\{\mathbf{j}_k^{\perp}\right\}(\mathbf{r}) = \gamma_k^{\perp}\mathbf{j}_k^{\perp} \quad \text{where} \quad \mathcal{T}_0^{\perp}\left\{\mathbf{w}\right\}(\mathbf{r}) = -\hat{\mathbf{n}}\times\hat{\mathbf{n}}\times\oint_{\partial\Omega} g_0\left(\mathbf{r}-\mathbf{r}'\right)\mathbf{w}\left(\mathbf{r}'\right)dS'.$$

Static Green function

Its spectrum has the following properties

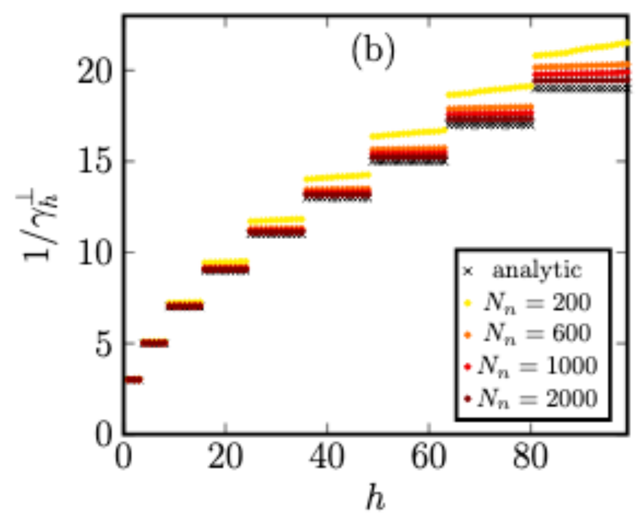
- The set of eigenvalues and the set of eigenfunctions are infinite countable
- The eigenvalues are real and positive
- The eigenfunctions associated to non-degenerate eigenvalues are orthogonal according to the scalar product

Static transverse current modes

- The eigenvalues and the modes depend on the shape of the particle, but are independent of both particle material and frequency of operation
- For a spherical surface of unit radius the eigenvalues have the following analytical expression

$$\frac{1}{\gamma_k^{\perp}} = 2n + 1.$$

The eigenvalues are compared with their analytical counterpart



Numerical generation of the static basis

The numerical auxiliary eigenvalue problems are respectively

- For longitudinal modes

$$\mathrm{T}_0^{\|\star\star}\mathbf{J}_h^\star = \gamma_h^\| \mathrm{R}^{\star\star}\mathbf{J}_h^\star;$$

$$(\mathrm{R}^{\star\star})_{pq} = \langle \mathbf{j}_p^\star, \mathbf{j}_q^\star \rangle$$

Longitudinal Eigenfunctions

Longitudinal Eigenvalues

- For transverse modes

$$\mathrm{T}^{\perp\circlearrowleft\circlearrowleft}\mathbf{J}_h^\circlearrowleft = \gamma_h^\perp \mathrm{R}^{\circlearrowleft\circlearrowleft}\mathbf{J}^\circlearrowleft$$

$$\left(\mathrm{R}^{\circlearrowleft\circlearrowleft}\right)_{pq} = \langle \mathbf{j}_p^\circlearrowleft, \mathbf{j}_q^\circlearrowleft \rangle$$

Transverse Eigenvalues

Transverse Eigenfunctions

Orthogonality and Gram matrices

If theoretically the mutual product between a transverse and a longitudinal static mode vanishes, this is verified only approximately numerically

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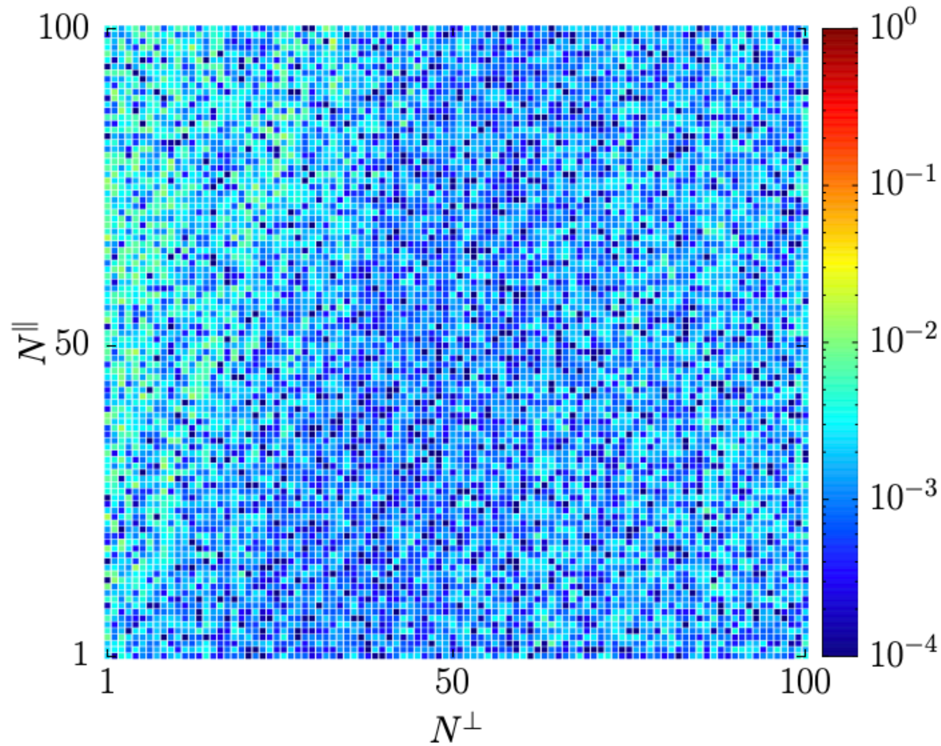
In order to test orthogonality of discrete longitudinal and transverse static modes the Gram matrix is evaluated.



Poggio-Miller-Chang-Harrington-Wu-Tsai Surface Integral Formulation

The effectiveness of the static base has been demonstrated through its use in the Galerkin projection of the PMCHWT Surface Integral Formulation

- Let us consider a linear, non-magnetic, homogeneous, isotropic object which occupies the volume Ω
- The object is illuminated by a time harmonic electromagnetic field
- The material has permittivity $\varepsilon^+(\omega)$ and permeability $\mu^+(\omega)$.
  It is surrounded by a background medium with permittivity $\varepsilon^-(\omega)$ and permeability $\mu^-(\omega)$

Poggio-Miller-Chang-Harrington-Wu-Tsai Surface Integral Formulation

The equivalent electric and magnetic surface current densities are solution of the following Integral problem

$$\mathcal{Z}\,\mathbf{J} = \mathbf{F},$$

where

EFIE operator

MFIE operator

- $\mathcal{Z} = \begin{pmatrix} \zeta^{-}\mathcal{T}_{-} + \zeta^{+}\mathcal{T}_{+} & \mathcal{K}_{-} + \mathcal{K}_{+} \\ -(\mathcal{K}_{-} + \mathcal{K}_{+}) & \mathcal{T}_{-}/\zeta^{-} + \mathcal{T}_{+}/\zeta^{+} \end{pmatrix}$
- $\mathbf{F} = [\mathbf{e}_0, \mathbf{h}_0]^{\mathsf{T}}$ is the excitation vector
- $\mathbf{J} = [\mathbf{j}_e, \mathbf{j}_m]^{\mathsf{T}}$ is the unknown vector composed by the surface currents

Galerkin projection

By representing

- any element of the space of transverse functions in term of the transverse static modes
- any element of the space of longitudinal functions in term of the longitudinal static modes

$$\begin{cases} \mathbf{j}_e(\mathbf{r}) \approx \sum_{p=1}^{N^\perp} \alpha_p^\perp \mathbf{j}_p^\perp(\mathbf{r}) + \sum_{q=1}^{N^\parallel} \alpha_q^\parallel \mathbf{j}_q^\parallel(\mathbf{r}), \\ \mathbf{j}_m(\mathbf{r}) \approx \sum_{p=1}^{N^\perp} \beta_p^\perp \mathbf{j}_p^\perp(\mathbf{r}) + \sum_{q=1}^{N^\parallel} \beta_q^\parallel \mathbf{j}_q^\parallel(\mathbf{r}). \end{cases}$$

the following discrete counterpart is obtained

$$\begin{pmatrix} \zeta^- \mathrm{T}_- + \zeta^+ \mathrm{T}_+ & \mathrm{K}_- + \mathrm{K}_+ \\ -(\mathrm{K}_- + \mathrm{K}_+) & \mathrm{T}_-/\zeta^- + \mathrm{T}_+/\zeta^+ \end{pmatrix}$$

$$\mathrm{ZJ} = \begin{pmatrix} \mathrm{E}_0 \\ \mathrm{H}_0 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{J}_e \\ \mathrm{J}_m \end{pmatrix}$$

$$\mathrm{E}_0 = \left(\mathrm{E}_0^\perp | \mathrm{E}_0^\parallel \right)^{\mathrm{T}}$$

$$\mathrm{H}_0 = \left(\mathrm{H}_0^\perp | \mathrm{H}_0^\parallel \right)^{\mathrm{T}}$$

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Galerkin projection

In this case, the discrete operators have a block structure with the following properties

$$\mathbf{T}_{\pm} = \left(\begin{array}{c|c} \mathbf{T}_{\pm}^{\perp,\perp} & \mathbf{T}_{\pm}^{\perp,\parallel} \\ \hline \mathbf{T}_{\pm}^{\parallel,\perp} & \mathbf{T}_{\pm}^{\parallel,\parallel} \end{array} \right) \qquad \mathbf{K}_{\pm} = \left(\begin{array}{c|c} \mathbf{K}_{\pm}^{\perp,\perp} & \mathbf{K}_{\pm}^{\perp,\parallel} \\ \hline \mathbf{K}_{\pm}^{\parallel,\perp} & \mathbf{K}_{\pm}^{\parallel,\parallel} \end{array} \right)$$

- Each matrix has $(N^{\parallel} + N^{\perp})$ x $(N^{\parallel} + N^{\perp})$ elements
- The static modes diagonalize static operators $\boldsymbol{\mathcal{T}}_0^{\perp}$ and $\boldsymbol{\mathcal{T}}_0^{\parallel}$

$$\left(\mathbf{T}_{\pm}^{\parallel\,\parallel}\right)_{pq} = \langle \mathbf{j}_p^{\parallel}, \boldsymbol{\mathcal{T}}_{\pm} \mathbf{j}_q^{\parallel} \rangle = \frac{\gamma_p^{\parallel}}{jk^{\pm}} \delta_{p,q} - jk^{\pm} \langle \mathbf{j}_p^{\parallel}, \boldsymbol{\mathcal{T}}_0^{\perp} \mathbf{j}_q^{\parallel} \rangle + \langle \mathbf{j}_p^{\parallel}, \boldsymbol{\mathcal{T}}_{d\pm} \mathbf{j}_q^{\parallel} \rangle$$

$$\left(\mathbf{T}_{\pm}^{\perp\,\parallel}\right)_{pq} = \langle \mathbf{j}_p^{\perp}, \boldsymbol{\mathcal{T}}_{\pm} \mathbf{j}_q^{\parallel} \rangle = \langle \mathbf{j}_p^{\perp}, \boldsymbol{\mathcal{T}}_{d\pm} \mathbf{j}_q^{\parallel} \rangle,$$

$$\left(\mathbf{T}_{\pm}^{\parallel\,\perp}\right)_{pq} = \langle \mathbf{j}_p^{\parallel}, \boldsymbol{\mathcal{T}}_{\pm} \mathbf{j}_q^{\perp} \rangle = \langle \mathbf{j}_p^{\parallel}, \boldsymbol{\mathcal{T}}_{d\pm} \mathbf{j}_q^{\perp} \rangle,$$

$$\left(\mathbf{T}_{\pm}^{\perp\,\perp}\right)_{pq} = \langle \mathbf{j}_p^{\perp}, \boldsymbol{\mathcal{T}}_{\pm} \mathbf{j}_q^{\perp} \rangle = -jk^{\pm} \gamma_h^{\perp} \delta_{p,q} + \langle \mathbf{j}_p^{\perp}, \boldsymbol{\mathcal{T}}_{d\pm} \mathbf{j}_q^{\perp} \rangle,$$

Result analysis- Silicon spherical particle

Relative error on the scattering efficiency

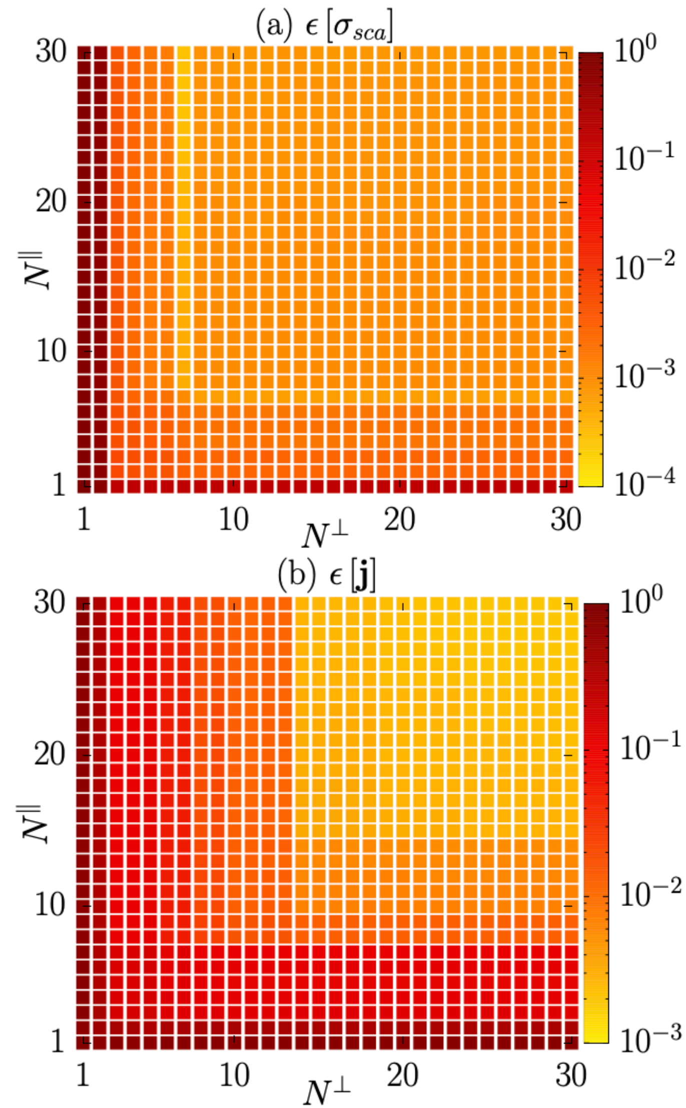
$$\epsilon\left[\sigma_{sca}\right] = \left|\sigma_{sca} - \tilde{\sigma}_{sca}\right| / \tilde{\sigma}_{sca}$$

Reference loop-star solution

Relative error on the equivalent surface current

$$\epsilon\left[\mathbf{J}\right] = \left\|\mathbf{J} - \tilde{\mathbf{J}}\right\|_2 / \left\|\tilde{\mathbf{J}}_e\right\|_2$$

Reference loop-star solution



- Symmetry with respect to the main diagonal
- With $N^{\parallel} = N^{\perp} = 3$: $\epsilon\left[\sigma_{sca}\right]$ < 0.01
- With $N^{\parallel} = N^{\perp} = 8$: $\epsilon\left[\mathbf{J}\right]$ < 0.1

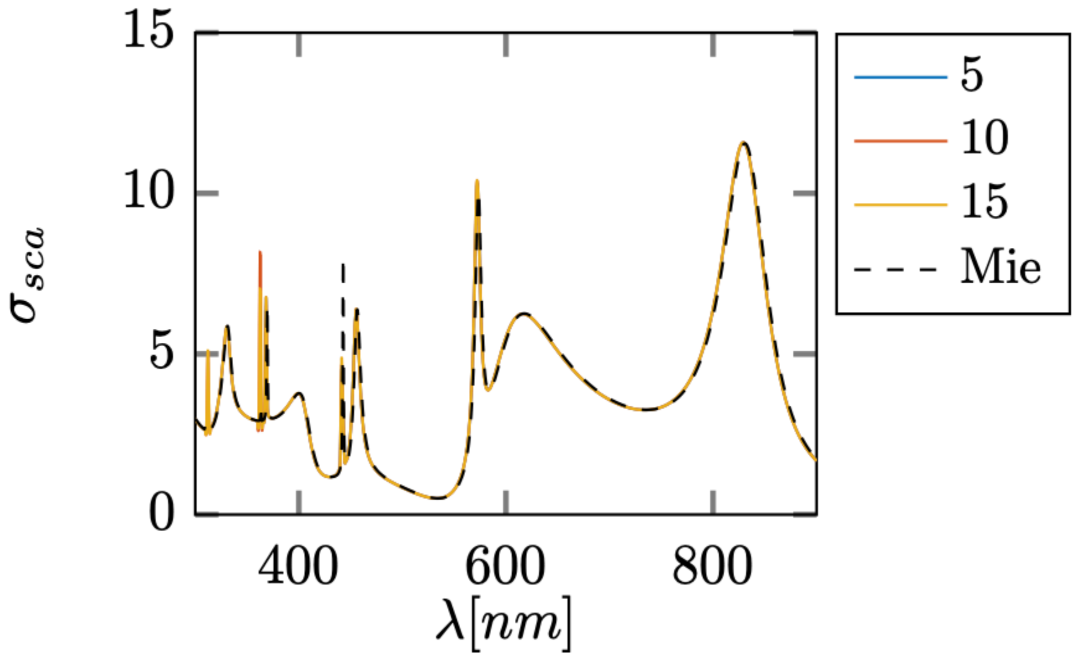
Result analysis- Silicon spherical particle

The scattering efficiency σ_{sca} as a function of the wavelength of the exciting, linearly polarized, plane wave

- Different solution using increasing truncation numbers $N^{\|} = N^{\perp}$ have been considered
- The analytic Mie solution has been used as reference (black dashed line)

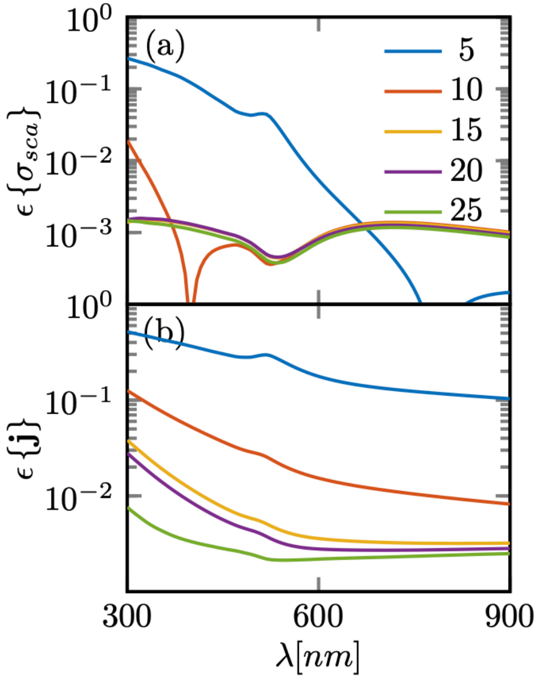
Low frequency →
With Truncation number =5
good agreement is ensured

High frequency →
With Truncation number =10
good agreement is ensured

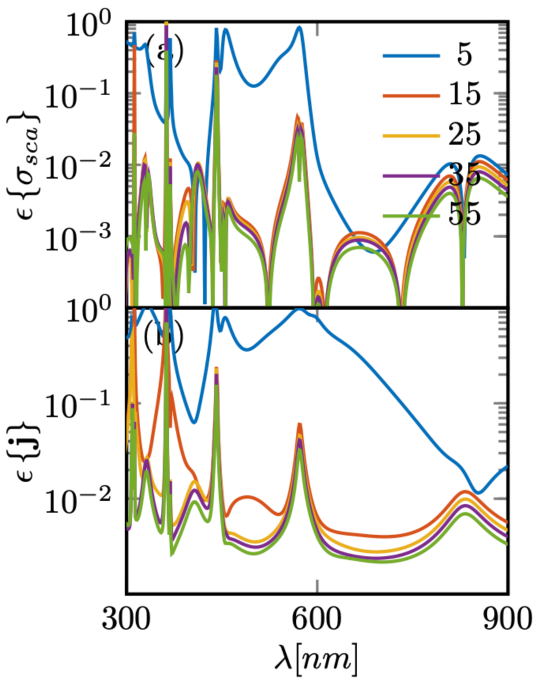


Result analysis- Sphere

Error in the evaluation of the scattering efficiency and of the equivalent surface currents as a function of the incident wavelength



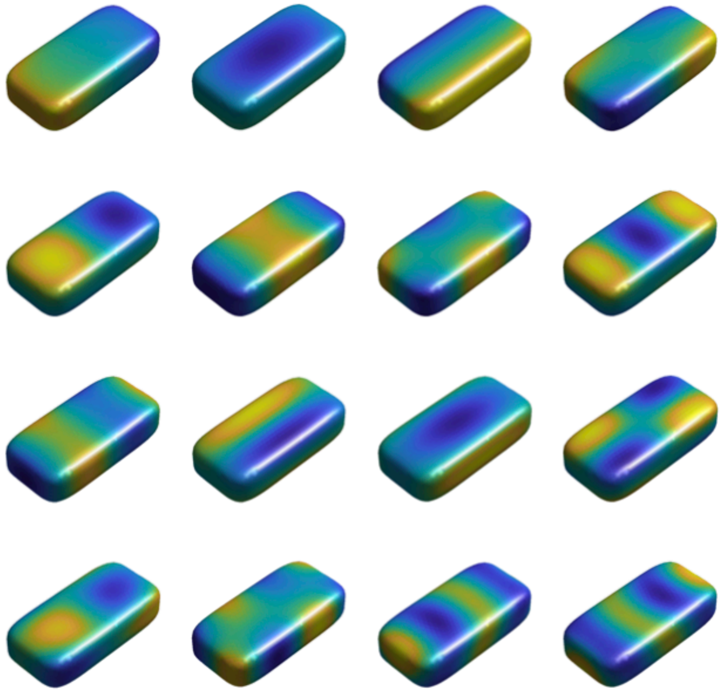
Gold sphere



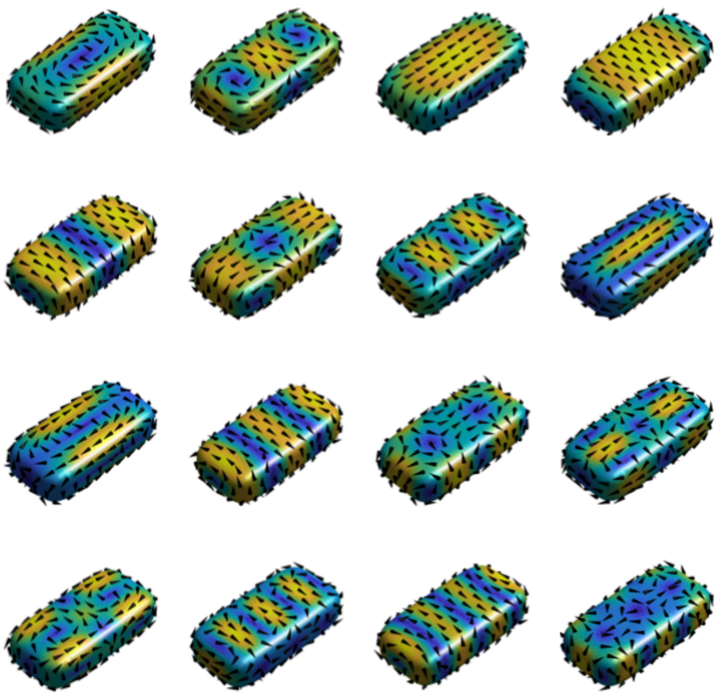
High permittivity sphere

Result analysis-Plasmon rod

Rod with semi-axis 1:0.5:0.25



The first 16 longitudinal static modes (sorted accordingly to their eigenvalue)



The first 16 transverse static modes (sorted accordingly to their eigenvalue)

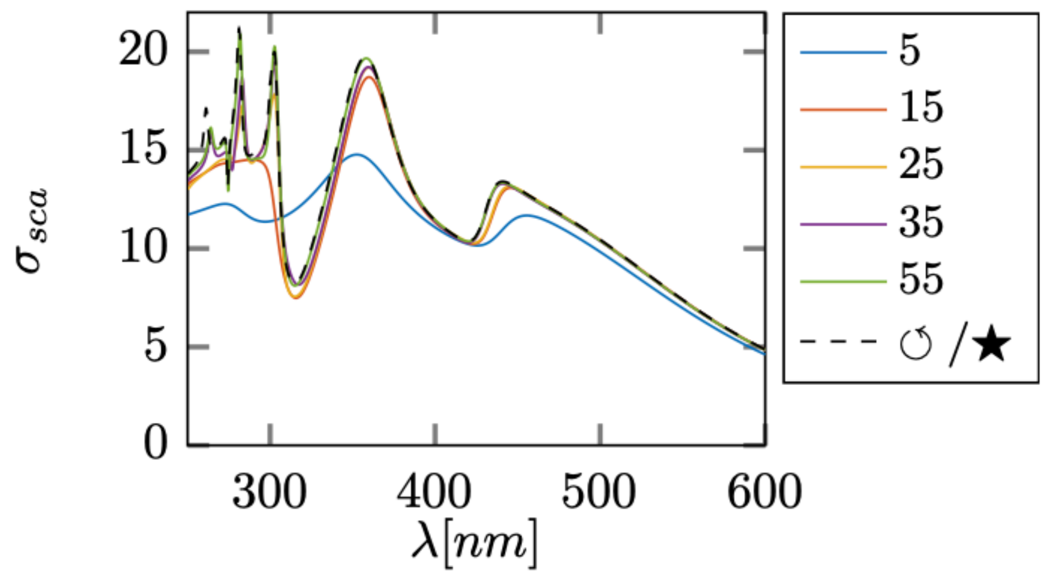
Result analysis- Dielectric rod

The scattering efficiency σ_{sca} as a function of the wavelength of the exciting, linearly polarized, plane wave

- Different solution using increasing truncation numbers $N^{\parallel} = N^{\perp}$ have been considered
- The loop-star solution has been used as reference (black dashed line)

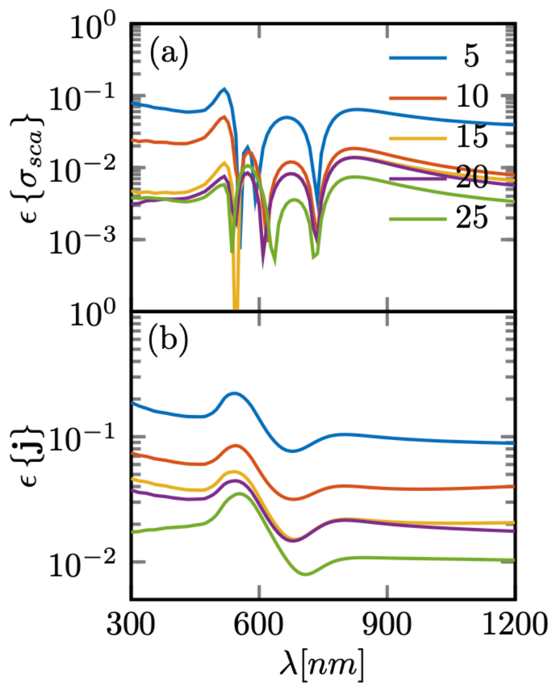
Wavelength >> dimension → With Truncation number =15 good agreement is ensured

Wavelength comparable→ With Truncation number =55 good agreement is ensured

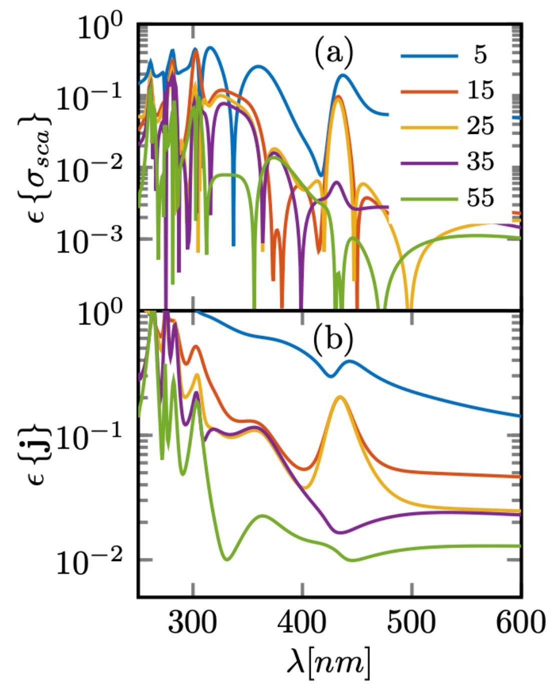


Result analysis- Plasmon rod

Error in the evaluation of the scattering efficiency and of the equivalent surface currents as a function of the incident wavelength



Gold rod



High permittivity rod

Conclusions

- For objects of individual size comparable to the wavelength of operation only few modes are sufficient to describe the emergent scattering response.
- The decomposition of the retarded Green function in the static Green function and a proper difference makes the integral operators diagonalized.
- The use of static modes combined with a suitable rescaling and rearranging of the unknowns makes the formulation immune to low-frequency breakdown.
- The static modes only depend on the shape of the object: the same static basis can be used regardless of the operating frequency and material of object → this fact enables the description of any scattering scenario involving one or more particles of a given shape in terms of the same “alphabet” of basis function.