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XXXIV Cycle - II year presentation

Static Transverse and Longitudinal Surface Modes as a Spectral Basis for Boundary Integral Methods

Our research group studies the electromagnetic properties of dielectrics, metals and semimetals at the nanoscale. We deal with arbitrary shape and size nanostructures in the full wave regime, providing both classical and quantum results. We perform theoretical and numerical analysis of the systems under investigation, trying to understand what the best ways to tackle the scattering problem are, in order to optimize the analysis and the design of custom nanostructures for photonic applications.

My personal main interest mostly lied in the development of new spectral method.
During my second year, a paradigma shift has been accomplished: from a local basis functions set (Rao- Wilton- Glisson basis functions) to solve Surface Integral Equation (SIE) Formulation , a new «global» set has been introduced. The global set is the union of two current-mode sets (Transverse Basis and Longitudinal Basis). A brief sketch is given below

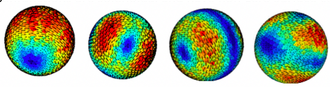
Transverse Basis

We consider the following auxiliary eigenvalue problem

$$\tau_o^{\perp}\{\mathbf{J}^{\perp}\}(\mathbf{r}) = -\gamma^{\perp}\mathbf{J}^{\perp}$$

Its spectrum has the following properties

- The set of eigenvalues $\{\gamma_k^{\perp}\}$end the set of eigenfunctions $\{\mathbf{J}_k^{\perp}\}$ are infinite countable;
- The eigenvalues and the eigenfunctions are real;
- The eigenvalues and the eigenfunctions depend on the shape of but not on its size;
- The eigenfunctions are solenoidal and have non-zero surface curl.



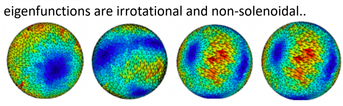
Longitudinal Basis

We consider the following auxiliary eigenvalue problem

$$\tau_o^{\parallel}\{\mathbf{J}^{\parallel}\}(\mathbf{r}) = -\gamma^{\parallel}\mathbf{J}^{\parallel}$$

Its spectrum has the following properties

- The set of eigenvalues $\{\gamma_k^{\parallel}\}$ end the set of eigenfunctions $\{\mathbf{J}_k^{\parallel}\}$ are infinite countable;
- The eigenvalues and the eigenfunctions are real;
- The eigenvalues and the eigenfunctions depend on the shape of but not on its size
- The eigenfunctions are irrotational and non-solenoidal..



Therefore, the surface current distributions can be represented as

$$\begin{cases} \mathbf{J}_e(\mathbf{r}) \simeq \sum_{p=1}^{N^{\perp}} \alpha_p^{\perp}\mathbf{J}_p^{\perp}(\mathbf{r}) + \sum_{q=1}^{N^{\parallel}} \alpha_q^{\parallel}\mathbf{J}_q^{\parallel}(\mathbf{r}) \\ \mathbf{J}_m(\mathbf{r}) \simeq \sum_{p=1}^{N^{\perp}} \beta_p^{\perp}\mathbf{J}_p^{\perp}(\mathbf{r}) + \sum_{q=1}^{N^{\parallel}} \beta_q^{\parallel}\mathbf{J}_q^{\parallel}(\mathbf{r}) \end{cases}$$

These two sets are orthogonal. A regular surface current density mode can be decomposed in terms of these two sets: surface Helmoltz decomposition. This basis has been used to represent both electric and magnetic surface currents in a full-wave surface integral formulation of the Maxwell's equation. It allows to algebrize the singular part of the involved full-wave integral operators.

Change of Basis

In order to achieve this aim effectively, a change to traditional Galerkin Method has been brough: discrete operators recurring in SIE formulations have been evaluated in the new Helmholtz Set by converting RWG discrete operators with suitable Transition Matrix .

Transversal Mode from RWG Basis Functions $\underline{J}_h^{\perp} = \sum_{q=1}^{N_t} \alpha_{h,q}^{\perp\bigcirc}\left(\sum_{s=1}^{N_t} \beta_{q,s}^{\bigcirc\mathrm{RWG}}\underline{W}_s\right) = \sum_{q=1}^{N_t}\sum_{s=1}^{N_t} \alpha_{h,q}^{\perp\bigcirc}\beta_{q,s}^{\bigcirc\mathrm{RWG}}\underline{W}_s$

Longitudinal Mode from RWG Basis Functions $\underline{J}_h^{\parallel} = \sum_{p=1}^{N_t} \alpha_{h,p}^{\parallel\star}\left(\sum_{r=1}^{N_t} \beta_{p,r}^{\star\mathrm{RWG}}\underline{W}_r\right) = \sum_{p=1}^{N_t}\sum_{r=1}^{N_t} \alpha_{h,p}^{\parallel\star}\beta_{p,r}^{\star\mathrm{RWG}}\underline{W}_r$

$$\underline{\underline{C}}^{\perp\mathrm{RWG}} = \underline{\underline{A}}^{\perp\bigcirc}\cdot\underline{\underline{B}}^{\bigcirc\mathrm{RWG}}$$
$$\underline{\underline{C}}^{\parallel\mathrm{RWG}} = \underline{\underline{A}}^{\parallel\star}\cdot\underline{\underline{B}}^{\star\mathrm{RWG}}$$

$$\begin{pmatrix} \underline{\underline{C}}^{\perp\mathrm{RWG}}\cdot\langle\underline{W}_r,\tau^{\parallel}\{\underline{W}_t\}\rangle\cdot\underline{\underline{C}}^{\perp\mathrm{RWGt}} & \underline{\underline{C}}^{\perp\mathrm{RWG}}\cdot\langle\underline{W}_r,\tau^{\parallel}\{\underline{W}_t\}\rangle\cdot\underline{\underline{C}}^{\parallel\mathrm{RWGt}} \\ \underline{\underline{C}}^{\parallel\mathrm{RWG}}\cdot\langle\underline{W}_r,\tau^{\parallel}\{\underline{W}_t\}\rangle\cdot\underline{\underline{C}}^{\perp\mathrm{RWGt}} & \underline{\underline{C}}^{\parallel\mathrm{RWG}}\cdot\langle\underline{W}_r,\tau^{\parallel}\{\underline{W}_t\}\rangle\cdot\underline{\underline{C}}^{\parallel\mathrm{RWGt}} \end{pmatrix}$$

$$\begin{pmatrix} \underline{\underline{C}}^{\perp\mathrm{RWG}}\cdot\langle\underline{W}_r,\tau^{\perp}\{\underline{W}_t\}\rangle\cdot\underline{\underline{C}}^{\perp\mathrm{RWGt}} & \underline{\underline{C}}^{\perp\mathrm{RWG}}\cdot\langle\underline{W}_r,\tau^{\perp}\{\underline{W}_t\}\rangle\cdot\underline{\underline{C}}^{\parallel\mathrm{RWGt}} \\ \underline{\underline{C}}^{\parallel\mathrm{RWG}}\cdot\langle\underline{W}_r,\tau^{\perp}\{\underline{W}_t\}\rangle\cdot\underline{\underline{C}}^{\perp\mathrm{RWGt}} & \underline{\underline{C}}^{\parallel\mathrm{RWG}}\cdot\langle\underline{W}_r,\tau^{\perp}\{\underline{W}_t\}\rangle\cdot\underline{\underline{C}}^{\parallel\mathrm{RWGt}} \end{pmatrix}$$

Italian Air Force

In my last PhD year, I'd like to apply this approach (Global Basis Functions Sets) to multi-particle Systems: in particular, by using what we achieved

- In the first year about Electromagnetic Modes and Resonances of a single particle
- In the second year about the composition of the Static Transverse and Longitudinal Surface Modes Set

Our aim will be to reduce complexity cost of a Multi-Particle Scattering problem.

UNIVERSITÀ DEGLI STUDI DI NAPOLI
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