PhD in Information Technology and Electrical Engineering

Università degli Studi di Napoli Federico II

PhD Student: Davide Fiore

XXIX Cycle

Training and Research Activities Report – Second Year

Tutor: Mario di Bernardo

1. Information

I received on 18/12/2014 the Laurea Magistrale cum laude in Ingegneria dell'Automazione from University of Naples "Federico II". Currently I'm a 2nd year PhD student of the XXIX cycle in ITEE.

Fellowship type: Fondo Sostegno ai Giovani - FSGDIS

Tutor: Prof. Mario di Bernardo

2. Study and Training activities

- **Courses**
  - "*Models, methods and software for Optimization*" (4 CFU)
    Lecturer: Antonio Sforza and Claudio Sterle
  - "*Local methods for nonlinear systems and control*" (3 CFU)
    Lecturer: Rodolphe Sepulchre
- **Seminars**
  - *"Mathematical modeling of atomic force microscopes"*
    Lecturer: Martin Homer
    Date: 22.04.2015 - 1h
  - *"Agents with truly perfect recall"*
    Lecturer: Nils Bulling
    Date: 28.04.2015 - 1h
  - *"On Abel differential equations of the 2nd kind and exact inversion of boost DC/AC"*
    Lecturer: Josep Olm
    Date: 13.05.2015 - 1h
  - *"Passivity-based control of nonlinear physical systems: a port-hamiltonian approach"*
    Lecturer: Alejandro Donaire
    Date: 27.05.2015 - 2h
  - *"Regularization of two-fold bifurcations in planar piecewise-smooth systems"*
    Lecturer: John Hogan
    Date: 23.06.2015 - 1h
  - *"Dynamics of asynchronous networks"*
    Lecturer: Mike Field
    Date: 19.02.2016 - 2h

3. Research activity

- **Title:**

  Incremental stability of Filippov systems via contraction theory

- **Research description:**

  Incremental stability has been established as a powerful tool to prove convergence in nonlinear dynamical systems [1]. It characterizes asymptotic convergence of trajectories with respect to one another rather than towards some attractor known a priori. Therefore, it is a useful alternative to the traditional Lyapunov functions approach when such a steady state solution is not known. Popular control applications include tracking and regulation, observer design, coordination, and synchronization.
  Several approaches to derive sufficient conditions for a system to be incrementally stable have been presented in the literature, among these contraction theory has been shown to be particularly effective [2], [3], [4]. A dynamical system is said to be contractive when the matrix measure of the Jacobian of the vector field is uniformly negative on the state space.

As the number of applications based on the use of switched and hybrid models increases, it is becoming increasingly important to characterize convergence and incremental stability of systems and networks modelled by ODEs with time-dependent or state-dependent discontinuities. Unfortunately, most of the results available in the literature on both incremental stability and contraction theory assume continuous differentiability of the system or network under investigation. Only few results deal with the problem of investigating incremental stability of piecewise-smooth (PWS), switched or hybrid dynamical systems.

In our recent work [5] in collaboration with Prof. S.J. Hogan, we extended contraction theory to the class of switched systems modelled by ODEs with discontinuous right-hand sides (or Filippov systems [6]). By using results on regularization of switched dynamical systems based on the singular perturbation theory [7], we derived sufficient conditions for convergence of any two trajectories of the Filippov system between each other within some region of interest. Specifically, a bimodal Filippov system is incrementally exponentially stable in a certain set if both modes of the system are contracting with respect the same norm and that the difference of the two modes evaluated at the switching manifold satisfies an additional condition. We then applied these conditions to the study of different classes of Filippov systems including piecewise smooth (PWS) systems, piecewise affine (PWA) systems and relay feedback systems. Furthermore, we showed that these conditions allow the system to be studied in metrics other than the Euclidean norm. The theoretical results have been illustrated by numerical simulations on a set of representative examples that confirm their effectiveness and ease of application.

As an application of the previous theoretical results we presented in [8] a switching control strategy to incrementally stabilize, either locally or globally, a class of nonlinear dynamical systems. Furthermore, based on these sufficient conditions, we proposed a design procedure to design a switched control action that is active only where the open-loop system is not sufficiently incrementally stable in order to reduce the required control effort.

Moreover, in collaboration with Dr. G. Russo, we are investigating how symmetries in ODEs describing the nodes' dynamics of a complex network can be exploited to induce in the network the emergence of the so-called *cluster synchronization*, that is when some groups of nodes of the network converge toward different solutions. Specifically, we proposed a design algorithm to select appropriate coupling functions to guarantee both bipartite synchronization and consensus [9].

[1] D. Angeli, *"A Lyapunov approach to incremental stability properties",* IEEE Transactions on Automatic Control, vol. 47, no. 3, pp. 410-421, 2002

[2] W. Lohmiller, J.J.E. Slotine, *"On contraction analysis for nonlinear systems",* Automatica, vol. 34, no. 6, pp. 638-696, 1998

[3] G. Russo, M. di Bernardo, E.D. Sontag, *"Global entrainment of transcriptional systems to periodic inputs"*, PLoS computational biology, vol. 6, no. 4, p. e1000739, 2010

[4] W. Wang, J.J.E. Slotine, *"On partial contraction analysis for coupled nonlinear oscillators"*, Biological cybernetics, vol. 92, no. 1, pp. 38-53, 2005

[5] M. di Bernardo, D. Fiore, S.J. Hogan, *"Contraction analysis of switched Filippov systems via regularization"*, submitted to Automatica

[6] V.I. Utkin, *Sliding modes in control and optimization*, Springer-Verlag, Berlin, 1992

[7] J. Sotomayor, M.A. Teixeira, *"Regularization of discontinuous vector fields",* International Conference on Differential Equations, Lisbon, 1996

[8] M. di Bernardo, D. Fiore, "*Switching control for incremental stabilization of nonlinear systems via contraction theory*", submitted to 2016 European Control Conference

[9] M. di Bernardo, D. Fiore, G. Russo *"On synchronization and consensus patterns in complex networks: from analysis to control"*, in preparation

- **Collaborations:**
  - Prof. John Hogan
    Department of Engineering Mathematics
    University of Bristol - UK
  - Dr. Giovanni Russo
    Optimization and Control Group
    IBM Research - Ireland

4. Products

- **Publications**

i. Already published:

- Mario di Bernardo, Davide Fiore, Giovanni Russo, Francesco Scafuti, *"Convergence, Consensus and Synchronization of Complex Networks via Contraction Theory"*, in "Complex Systems and Networks: Dynamics, Controls and Applications", J. Lu, X. Yu, G. Chen and W. Yu editors, p. 313-339, Springer Berlin Heidelberg, 2016

ii. Submitted:

- M. di Bernardo, D. Fiore, S.J. Hogan, *"Contraction analysis of switched Filippov systems via regularization"*, submitted to *Automatica* on 27/07/2015, revised version resubmitted on 24/02/2016 http://arxiv.org/abs/1507.07126
- M. di Bernardo, D. Fiore, *"Switching control for incremental stabilization of nonlinear systems via contraction theory"*, submitted to *2016 European Control Conference* on 20/10/2015 http://arxiv.org/abs/1510.08368

iii. In preparation:

- M. di Bernardo, D. Fiore, G. Russo *"On synchronization and consensus patterns in complex networks: from analysis to control"*

5. Conferences and Seminars

- Poster presented during the conference
  "Open Problems in Nonsmooth Dynamics"
  Centre de Recerca Matemàtica
  Universitàt Autònoma de Barcelona
  Barcelona (Spain) – 01-05.02.2016
  http://www.crm.cat/en/Activities/Curs_2015-2016/Pages/CNonsmooth.aspx

6. Activity abroad

- Two (2) periods of research at the Department of Engineering Mathematics of University of Bristol (UK) to collaborate with Prof. S.J. Hogan.
  From 25.11.2015 to 05.12.2015 and from 15.02.2016 to 20.02.2016

7. Tutorship

- Assistant for exercises of the Laurea course "Controlli automatici" (Cod. 02826), held by Prof. Mario di Bernardo, 11 hours.
- Assistant for exercises of the Laurea Magistrale course "Dinamica e Controllo Non Lineare" (Cod. 17066), held by Prof. Mario di Bernardo, 6 hours.

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Cycle XXIX

	Credits year 1							Credits year 2							Credits year 3									
	1 bimonth	2 bimonth	3 bimonth	4 bimonth	5 bimonth	6 bimonth	**Summary**	1 bimonth	2 bimonth	3 bimonth	4 bimonth	5 bimonth	6 bimonth	**Summary**	**Estimated**	1 bimonth	2 bimonth	3 bimonth	4 bimonth	5 bimonth	6 bimonth	**Summary**	**Total**	**Check**
Modules	0	13	3	3	0	0	**19**	3	4	0	0	0	0	**7**	**10**							**0**	**26**	**30-70**
Seminars	1	1	0	0.2	1.4	1	**4.6**	0.4	0.8	0	0	0	0.4	**1.6**	**4**							**0**	**6.8**	**10-30**
Research	9	0	6	7	10	9	**41**	9	8	10	8	8	5.8	**48.8**	**43**							**0**	**89.8**	**80-140**
	10	13	9	10.2	11.4	10	**65.6**	12.4	12.8	10	8	8	6.2	**57.4**	**57**	0	0	0	0	0	0	**0**	**123**	**180**