Algorithm for Computing Domains of Attraction in Planar Switched Linear Systems

Christian Erazo
Tutor: Mario di Bernardo

Università degli Studi di Napoli Federico II
PhD in Information Technology and Electrical Engineering

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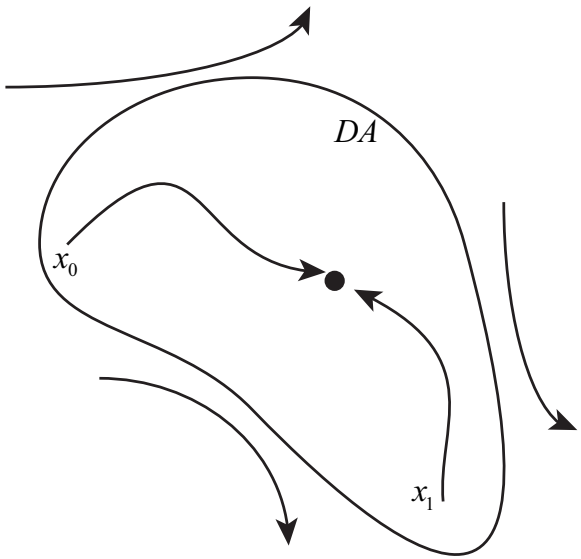
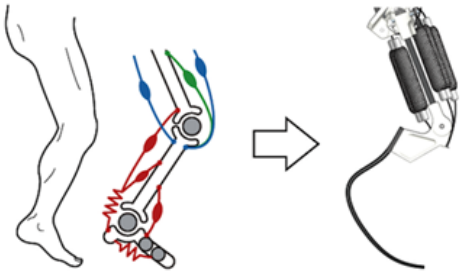
DA - Switched Linear Systems

Domain of attraction (DA): The set of initial conditions corresponding to trajectories that converge towards an equilibrium point.





Objective

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- Estimating domains of attraction in planar switched linear systems.

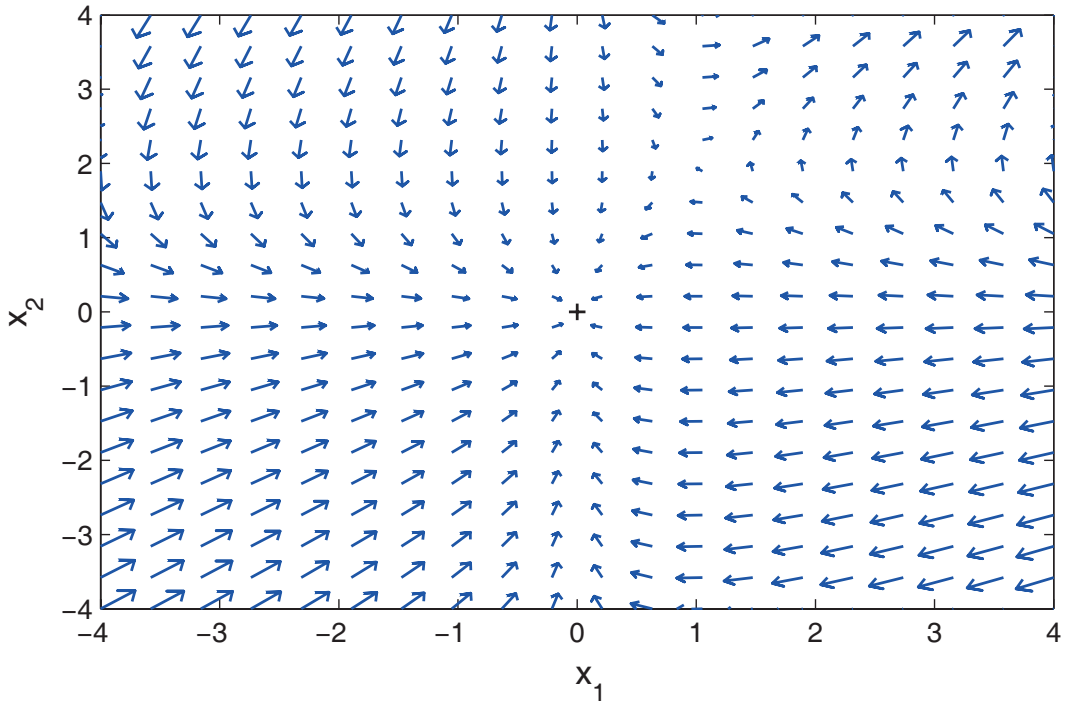
[1] F. Blanchini and S. Miani, Set-theoretic methods in control, Springer, 2007.

[2] T. Alamo et al., A new concept of invariance for saturated systems, Automatica, 2006.

[3] J. Mancilla et al., An extension of LaSalles invariance principle for switched systems, Springer, 2007.

Methodology

The approach consider an initial estimate provided by a polyhedral function $V(x)$, then the set is decomposed and enlarged by verifying the direction of the vector field along the boundary of the set.



Algorithm 1 Computation of domain of attraction of constrained switched systems

Input : $f(x)$ and $\wp$.

Output : $\wp^{max}$ (Domain of attraction).

1. Let

$$\wp = \{x : Wx \leq 1\}$$

2. Shift every line segment of the set along its normal direction and compute the new segments

$$s_N = (1-t)P_0 + tP_1$$

3. Verify

$$s_i \cdot f(x) < 0 \tag{1}$$

if (1) is met, go to step 2., else set $\wp = \wp^{max}$

Example

To illustrate the methodology, we consider a single input continuous-time saturated system described by:

$$\begin{aligned} \dot{x} &= Ax + B\text{sat}(u) \\ u &= Fx \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input and $\text{sat}(.) : \mathbb{R} \to \mathbb{R}$ is the saturation function.

[4] Zhao et al., Estimation of the domain of attraction for asymmetric saturated linear systems via Polyhedral Lyapunov Functions, 10th World Congress on Intelligent Control and Automation (WCICA), 2012.

Approaches

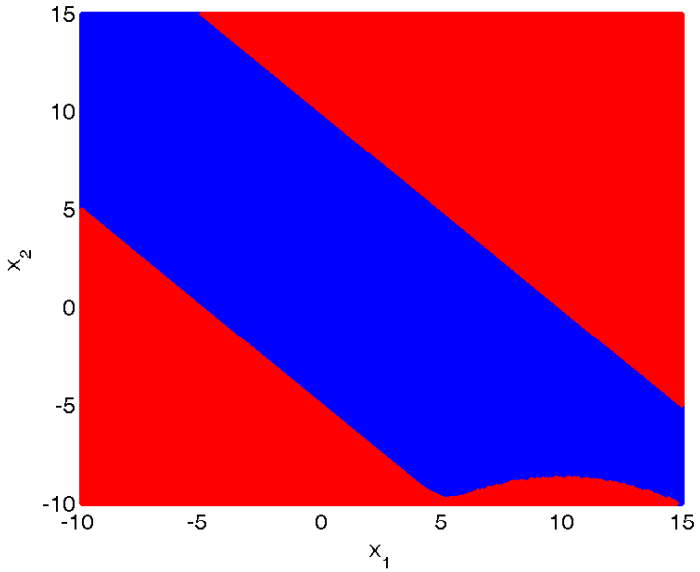


Figure 1: by brute force: blue region–stable

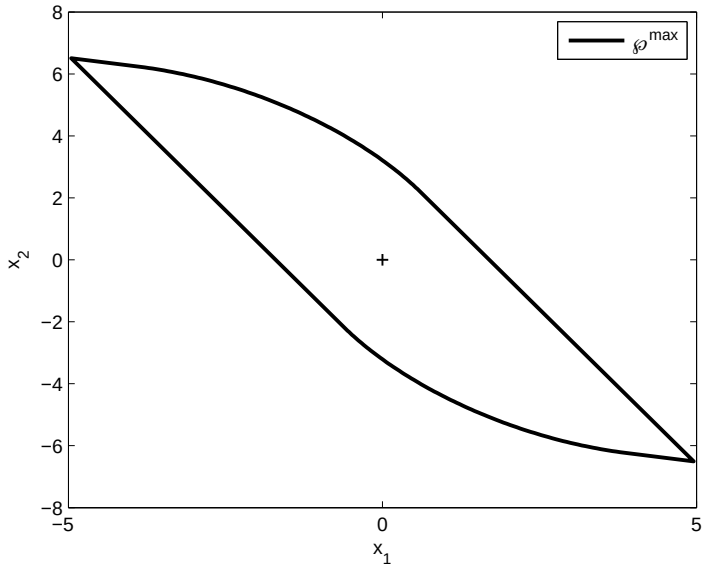
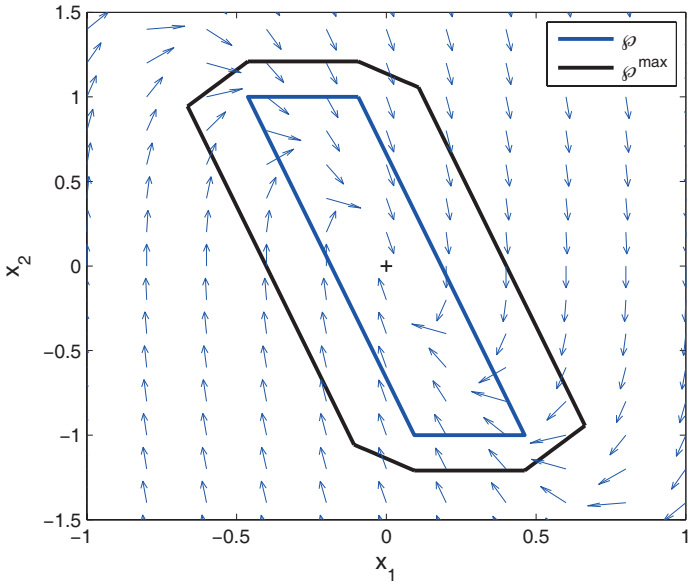
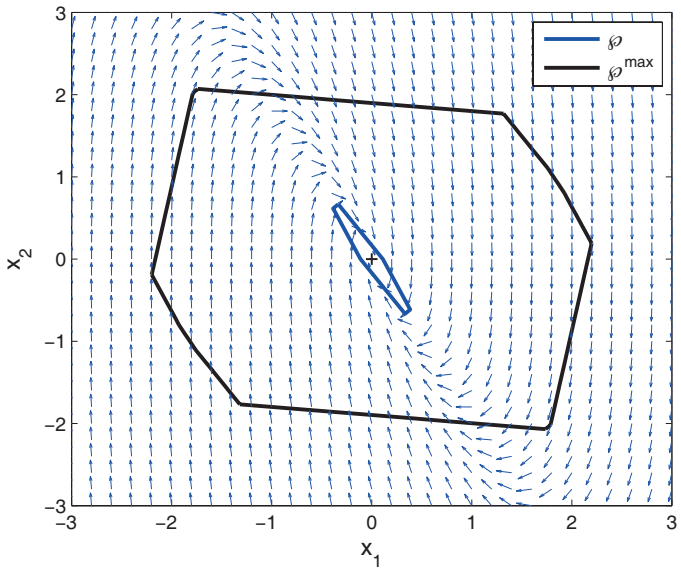


Figure 2: by polyhedral Lyapunov functions

[5] F. Blanchini and S. Miani, Set-theoretic methods in control, Springer, pp.295–305, 2007.

Our approach





The simulations show the effectiveness of our strategy, especially when the number of sides of the initial set is increased.

Conclusions

1. Literature review of several methodologies to compute domains of attraction in switched linear systems.
2. We observed that it is possible to estimate numerically the invariant region around an equilibrium by analyzing how the vector field direction changes along the boundaries of an initial set.
3. We proposed an approach to compute numerically domains of attraction in saturated switched systems.

Future Work

1. To extend this study to switched systems with discontinuities in the control law.
2. To include a better an more refined estimate of the region of attraction by considering different methodologies to iteratively expand the region of interest.

Training Activities

Courses:

- Prof. John Hogan, Theory and applications of piecewise smooth systems.
- Prof. Stephen Boyd, Convex Optimization.
- Prof. Henning Schulzrinne, Three core issues for the Internet: things, security and economics.

Seminars:

- Seminars of research group SINCRO.

Total credits: 10

Thanks for your attention!!