



PhD in Information Technology and Electrical Engineering

Università degli Studi di Napoli Federico II

PhD Student: Christian Erazo Ordoñez

XXIX Cycle

Training and Research Activities Report – Third Year

Tutor: Mario di Bernardo



1. Information

I received the Electronic Engineering and Master of Industrial Automation degree from National University of Colombia on 2010 and 2012 respectively. I belong to PhD in Information Technology and Electrical Engineering, Cycle 29°. Currently, I am receiving support from a fellowship provided by Università degli Studi di Napoli Federico II. My research activities are supervised by Professor Mario di Bernardo.

2. Study and Training Activities

In the third year, I followed some courses to improve my knowledge in nonlinear dynamics and hybrid systems.

a) Courses

- Name: Prof. Andrew Teel, Introduction to stochastic hybrid dynamical systems.
Location: Univ. of Trento, Trento, Italy.
Date: 1 May 2016.
Credits: 3,75.
- Name: Prof. S. J. Hogan, Delay Differential Equations
Location: Univ. of Naples Federico II, Naples, Italy.
Date: 17 May 2016.
Number of Hours: 18.
Credits: 3.

b) Seminars

- Name: Dynamics of asynchronous networks.
Lecturer: Dr. Christian Bick (College and Engineering, Mathematics and Physical Science of University of Exeter)
Date: 19 Feb 2016.
Number of Hours: 1
Credits: 0.2
- Name: Reactive power control in AC networks, from the state of the art to the Chopper controlled impedance concept.
Lecturer: Dr. Philippe Ladoux (Institut National Polytechnique de Toulouse, France)
Date: 31 January 2017
Number of Hours: 1
Credits: 0.4.

- Name: On differential inclusions with maximal monotone operators
Lecturer: Dr. Luigi Iannelli (University of Sannio, Benevento, Italy)
Date: 13 Febrero 2016
Number of Hours: 1
Credits: 0.4.
- Name: Contraction analysis of switched systems
Lecturer: PhD student, Davide Fiore (University of Naples, Federico II, Naples, Italy)
Date: 13 February 2016
Number of Hours: 1
Credits: 0.4.
- Name: Averaging with state jumps
Lecturer: Dr. Luigi Iannelli (University of Sannio, Benevento, Italy)
Date: 13 February 2016
Number of Hours: 1
Credits: 0.4.
- Name: Multi agent coordination with event-triggered cloud support
Lecturer: Dr. Davide Liuzza (University of Sannio, Benevento, Italy)
Date: 13 February 2016
Number of Hours: 1
Credits: 0.4.
- Name: Control of Multiplex Networks
Lecturer: Dr. Mario di Bernardo (University of Naples, Federico II, Naples, Italy)
Date: 13 February 2016
Number of Hours: 1
Credits: 0.4.
- Name: Coopetition and cooperostiy in dynamic social networks
Lecturer: Dr. Francesco Vasca (University of Sannio, Benevento, Italy)
Date: 13 February 2016
Number of Hours: 1
Credits: 0.4.
- Name: Complex networks modeling of financial markets
Lecturer: Dr. Piero De Lellis (University of Naples, Federico II, Naples, Italy)
Date: 13 February 2016
Number of Hours: 1
Credits: 0.4.

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- Name: Systems biology
Lecturer: PhD student, Amol Yerudkar (University of Sannio, Benevento, Italy)
Date: 13 February 2016
Number of Hours: 1
Credits: 0.4.
- Name: Parking optimization
Lecturer: PhD student, Ali Forootani (University of Sannio, Benevento, Italy)
Date: 13 February 2016
Number of Hours: 1
Credits: 0.4.

Training and Research Activities Report – Third Year

PhD in Information Technology and Electrical Engineering – XXIX Cycle

Christian Erazo Ordoñez

Student: Christian Erazo christian.erazoordonez@unina.it		Tutor: Mario di Bernardo mario.dibernardo@unina.it		Cycle XXIX																						
		Credits year 1		Credits year 2		Credits year 3																				
	Estimated	1	2	3	4	5	6	Summary	Estimated	1	2	3	4	5	6	Summary	Estimated	1	2	3	4	5	6	Summary	Total	Check
Modules	20		8				2	10	15			14				14	21		6,8					6,8	31	30-70
Seminars	8						2	2	6	0,4	0,6				0,6	1,8	3,4	12	0,2			0,2	4,4	4,8	10	10-30
Research	32	9	7	6	7	7	6	42	39	7	5	7	8	4	8	39	30	9	9	6	7	4	8	43	124	80-140
	60	9	15	6	7	7	10	54	60	7,4	20	7	8	4,6	9,8	56	63	9,2	16	6	7	4,2	12	55	165	

Year	Lecture/Activity	Type	Credits	Certification	Notes
1	Theory and applications of piecewise smooth systems.	Course	5	x	
1	Convex Optimization.	Course	3	x	
1	Three core issues for the Internet: things, security and economics.	Course	2	x	
1	Seminars of research group SINCRO	Seminar	2	x	
2	Corso di Italiano	Course	6	x	
2	Dinamica e Controllo Nonlineare.	Course	6	x	
2	Models, methods and software for Optimization.	Course	4	x	
2	Passivity-based control of nonlinear physical systems: a port-hamiltonian approach	Seminar	0,4	x	
2	Mathematical Modelling of Atomic Force Microscopes.	Seminar	0,2	x	
2	On Abel differential equations of the 2nd kind and exact inversion of boost DC/AC converters.	Seminar	0,2	x	
2	Regularization of two-fold bifurcations in planar piecewise-smooth systems.	Seminar	0,2	x	
2	Stochastic dynamics interrupted with large jumps at random times.	Seminar	0,2	x	
2	Analysis and design of genetic control circuits for metabolism.	Seminar	0,2	x	
2	Mathematical Modelling of the Steroidogenic Gene Regulatory Network in the Adrenal Gland.	Seminar	0,2	x	
2	CMOS smart gas sensors, temperature sensors and IR devices.	Seminar	0,6	x	
2	Seminars of research group SINCRO	Seminar	1	x	
3	Dynamics of Asynchronous Networks	Seminar	0,2	x	
3	Reactive Power Control in AC Networks	Seminar	0,2	x	
3	Delay differential equations (DDEs)	Course	3	x	
3	Introduction to stochastic hybrid dynamical systems	Course	3,75	x	
3	Piecewise smooth systems	Seminar	0,2	x	
3	Seminars of research group SINCRO	Seminar	1	x	
3	On differential inclusions with maximal monotone operators	Seminar	0,4	x	
3	Contraction analysis of switched systems	Seminar	0,4	x	
3	Averaging with state jumps	Seminar	0,4	x	
3	Multi agent coordination with event-triggered cloud support	Seminar	0,4	x	
3	Partial Control and Observation of Complex Networks	Seminar	0,4	x	
3	Control of Multiplex Networks	Seminar	0,4	x	
3	Coopetition and cooperostiy in dynamic social networks	Seminar	0,4	x	
3	Complex networks modeling of financial markets	Seminar	0,4	x	
3	Systems biology	Seminar	0,4	x	
3	Parking optimization	Seminar	0,4	x	

3. Research activity

Title: Dynamics of coupled mechanical systems with friction

The scope of my doctoral activity concerns with the study of dynamics of systems with discontinuous vector fields, as a remarkable example we consider coupled systems with friction. Complex phenomena such as stick-slip vibrations excited by friction, chaos and self-organized behavior are a common phenomenon underlying the behavior of several mechanical systems with friction. Some applications include formation of traffic jams in a single-lane highway traffic [1], distribution of earthquakes [2], suspension dynamics in vehicles [3] among others. Therefore, understanding the main features underlying the behavior of single and coupled discontinuous systems is of great importance in many practical applications.

One particular problem in discontinuous systems is the computation of the basins of attraction. Important information about complex behavior caused by friction impacts or damping, useful in the design of mechanical devices are provided by basins of attraction. During the first year of my PhD, we reviewed several methods for computing basins of attraction (BA) in Filippov systems [5,8]. The problem of computing basins of attractions in switching systems is mostly addressed by Lyapunov methods in the context of control theory, where regions of attraction are estimated as sublevel sets of a given Lyapunov function. However, this method provides conservative results meaning that the estimated region is smaller than the exact basin of attraction [8]. Cell mapping methods (CM) provide a computationally efficient way to analyze the long-term behavior of dynamical systems [11]. Their key characteristic is the approximation of the continuous state space via a discrete array of cells known as cell-state space. Then, a cell-to-cell map is created by performing short-time integrations, from the center of each cell, to the cell which contains the endpoint of the trajectory.

Fewer results using cell mapping methods have been reported in discontinuous systems with sliding solutions, mainly due to the fact that standard integration routines are inaccurate or inefficient, or both, in the region where discontinuities in the derivatives occur. For example, a possible source of numerical problems is the presence of small oscillations around the discontinuity boundary (numerical chattering) that may arise during sliding. A disadvantage of existing algorithms based on cell-to-cell mapping is the fact that the region of interest is pre-defined by the user which implies that extra computations are required if it is desired to explore a different region of state space [8,10,11]. Parallel processing capabilities of modern architectures have also been exploited, in the case of smooth and high order systems to consider different cell dimensions and several refinement stages within cell mapping methods. However, these techniques have not been extended, as far as we are aware, for discontinuous systems.

Therefore, with the collaboration of Professors Martin Homer and Petri Piiroinen we developed an algorithm based on the Simple Cell Mapping (SCM) method which exploits

the event-driven integration routine proposed in [12], that can cope with the presence of sliding solutions and automatically correct for possible numerical drifts. Our algorithm encompasses a dynamic selection of the cells. Specifically, after an initial application of SCM, layers of cells are added and examined iteratively. The mapping information is stored and used at each iteration, such that integrations for just the extra cells are performed. Moreover, a refinement stage is used to obtain a better resolution of the basin boundary. We illustrate the effectiveness of our algorithm by computing basins of attraction for Relay feedback systems, Sliding Control systems and non-smooth systems as reported in [13].

The implementation of the numerical tool for the computation of the basins of attractions in bimodal piecewise systems was based on the full characterization of the entry and exit points of the sliding flows, existing in the literature of PWS systems [4,5,16]. In the case of a single discontinuity manifold of co-dimension 1, Filippov formalism has provided a widely accepted mathematical framework to understand the dynamics on the discontinuity surface [4]. However, when we consider high order discontinuity surfaces, an ambiguity arises in the construction of the sliding vector field. This problem has been extensively studied see for example [14]. In the case of Filippov systems of co-dimension 2, there exist in the literature two systematic proposals to avoid the ambiguity in the Filippov convex method, they are the bilinear combination and a recent approach called the moments method. A nonlinear formulation to construct the sliding vector field called the *Hidden dynamics* has been presented in [15]. This formulation has shown to be effective in modeling real mechanical phenomena like stiction, not captured by applying Filippov's method [4, 15]. In [15] the authors investigate how the regularization of the discontinuous systems can be extended to the nonlinear sliding vector fields, while in [16] and [17] the authors perform an analysis of bifurcation of the hidden dynamics and also illustrate the strange effects induced by the nonlinear dynamics. In the second year of my PhD, during my visit to the Department of Engineering Mathematics at University of Bristol, with the collaboration of Professor Martin Homer and the PhD student Emmanuel Lorenzano, we studied the dynamics of two coupled oscillators in which we modeled the friction force via the hidden dynamics approach. This analysis provided a full characterization of the nonlinear sliding dynamics of co-dimension 1 and higher order sliding modes, and more importantly the ambiguity in selecting the nonlinear sliding vector field in the co-dimension 2 surface is resolved by using the regularization approach.

During the last year we studied the dynamics of multiple coupled oscillators from the point of view of synchronization, where the goal is that all states of oscillator in the network, converge towards each other. Examples of networks of piecewise dynamical systems can be found in biochemical reactions, power grids and arrays of mechanical oscillators with friction [1-3,18]. It is therefore of great importance to derive conditions to guarantee synchronization in networks of discontinuous systems. Currently, most of the literature focuses on networks with switching topologies. The problem of considering networks in which each agent is described by piecewise system is challenging and some preliminary results have been proposed in the literature. According to this, we performed an extensive numerical analysis for studying synchronization in chaotic friction oscillators, characterizing the influence of dynamic coupling and providing an estimation of the

synchronization region in terms of the coupling parameters. Initially, we consider the simple case of two coupled oscillators, then we extend the analysis to the case of larger networks of coupled systems with different network topologies. Moreover, preliminary analytical results of the convergence of a network of N friction oscillators based on contraction analysis are presented. The obtained results are validated through a representative example.

References:

- [1] A. Járai-Szabó, F., & Néda, Z., Earthquake model describes traffic jams caused by imperfect driving styles, *Physica A: Statistical mechanics and its applications*, 391(22) : 5727-5738, 2012.
- [2] Burridge, R. & Knopoff, Leon, Model and theoretical seismicity, *Bulletin of the seismological society of america*, 1967.
- [3] Armstrong-Hélouvry, Brian and Dupont, Pierre and De Wit, Carlos Canudas, A survey of models, analysis tools and compensation methods for the control of machines with friction, *Automatica*, vol. 30, Elsevier, 1994.
- [4] A. F. Filippov and F. M. Arscott, *Differential equations with discontinuous right-hand sides: control systems*, vol. 18, Springer Verlag, 1988.
- [5] P. Kowalczyk and P.T. Piiroinen. Two-parameter sliding bifurcations of periodic solutions in a dry-friction oscillator. *Physica D: Nonlinear Phenomena*, 237(8):1053 – 1073, 2008.
- [6] M Bernardo, C Budd, AR Champneys, P Kowalczyk, *Piecewise-smooth dynamical systems: theory and applications*.
- [7] C. K. Luk, G. Chesi, and D. Han, “Guaranteed estimates of the domain of attraction for a class of hybrid systems,” in *IEEE 52nd Annual Conference on Decision and Control (CDC)*, Dec 2013, pp. 2024–2029.
- [8] M. Demenkov, “Estimating basin of attraction in piecewise-linear systems by nonsmooth lyapunov functions,” in *International Meeting on Analysis and Applications of Non-smooth Systems*, 2014.
- [9] Hsu, C. S., *Cell-to-cell mapping: a method of global analysis for nonlinear systems*, vol. 64, Springer Verlag 1987.
- [10] Joanna F. Mason, Petri T. Piiroinen, R. Eddie Wilson, and Martin E. Homer, Basins of attraction in nonsmooth models of gear rattle, *International Journal of Bifurcation and Chaos*, pp. 203–224. 2009.
- [11] Gyebrószki, G., and Csernák, G., *Methods for the Quick Analysis of Micro-chaos*. In *Applied Non-Linear Dynamical Systems* (pp. 383-395). Springer International Publishing. (2014).
- [12] Petri T Piiroinen and Yuri A Kuznetsov, An event-driven method to simulate filippov systems with accurate computing of sliding motions, *ACM Transactions on Mathematical Software (TOMS)*, 34,(2008), p. 13.
- [13] M. di Bernardo, C. Erazo, M. Homer and P. Piiroinen, Dynamic cell-to-cell mapping algorithm for computing basins of attraction in bimodal Filippov Systems, *Conference on open problems in nonsmooth dynamics*, Barcelona, Spain.

- [14] Dieci, L., & Difonzo, F., A comparison of Filippov sliding vector fields in codimension 2. *Journal of Computational and Applied Mathematics*, 262, 161-179, 2014.
- [15] Jeffrey, M. R. (2014). Hidden dynamics in models of discontinuity and switching. *Physica D: Nonlinear Phenomena*, 273, 34-45.
- [16] Jeffrey, M. R. (2016). Hidden Bifurcations and Attractors in Nonsmooth Dynamical Systems. *International Journal of Bifurcation and Chaos*, 26(04), 1650068.
- [17] Jeffrey, M. R. (2015). Smoothing tautologies, hidden dynamics, and sigmoid asymptotics for piecewise smooth systems. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 25(10), 103125.
- [18] Leine, R., & Nijmeijer, H. (2013). *Dynamics and bifurcations of non-smooth mechanical systems (Vol. 18)*. Springer Science & Business Media.

Collaborations:

- Prof. Martin Homer, Faculty of Engineering University of Bristol.
- Dr. Petri Piiroinen, Faculty of University of Galway, Ireland.
- PhD student Emanuel Lorenzano, University of Bologna.

4. Products

a) Publications

I. Articles

- C. Erazo, M. Homer, P. Piiroinen and M. di Bernardo, Extended Simple Cell Mapping for Filippov Systems, *International Journal and Bifurcations (IJBC)*, Feb 2017 (Submitted).

II. Articles in preparation

- C. Erazo, E. Lorenzano, M. Homer and M. di Bernardo, Dynamics and synchronization of coupled friction oscillators.

5. Conference and seminars

Title: Dynamic cell-to-cell mapping algorithm for computing basins of attraction in bimodal Filippov Systems.

Event: Conference on open problems in nonsmooth dynamics.

Location: Centre de Recerca Matemàtica (CRM), Barcelona, Spain.

Date: February 1 to 5, 2016.

http://www.crm.cat/en/Activities/Curs_2015-2016/Pages/CNonsmooth.aspx

6. Activity Abroad

Università degli Studi di Napoli Federico II

Period of research at the Department of Engineering Mathematics of University of Bristol (United Kingdom) to collaborate with Prof. Martin Homer. From 13.01.2016 to 12.04.2016.

7. Tutorship

- Assistant for exercises of the B.Sc. course “Controlli automatici”, held by Prof. Mario di Bernardo, 4 hours.
- Assistant for exercises of the B.Sc. course “Dinamica e Controllo nonlineare”, held by Prof. Mario di Bernardo, 8 hours.