



PhD in Information Technology and Electrical Engineering

Università degli Studi di Napoli Federico II

PhD Student: Christian Erazo Ordoñez

XXIX Cycle

Training and Research Activities Report – Second Year

Tutor: Mario di Bernardo



1. Information

I received the Electronic Engineering and Master of Industrial Automation degree from National University of Colombia on 2010 and 2012 respectively. I belong to PhD in Information Technology and Electrical Engineering, Cycle 29°. Currently, I am receiving support from a fellowship provided by Università degli Studi di Napoli Federico II. My research activities are supervised by Professor Mario di Bernardo.

2. Study and Training Activities

In the second year I followed some courses to improve my knowledge in nonlinear dynamics and optimization techniques.

a) Courses

- Name: Prof. Mario di Bernardo, Dinamica e Controllo Nonlineare
Location: Univ. of Naples Federico II, Naples, Italy.
Date: March 2015.
Credits: 6.
- Name: Prof. Antonio Sforza and Claudio Sterle, Models, methods and software for Optimization.
Location: Univ. of Naples Federico II, Naples, Italy.
Date: March 2015.
Number of Hours: 18.
Credits: 4.

b) Seminars

- Name: Mathematical Modelling of Atomic Force Microscopes.
Lecturer: Dr. Martin Homer
Date: 22 April 2015.
Number of Hours: 1
Credits: 0.2
- Name: On Abel differential equations of the 2nd kind and exact inversion of boost DC/AC converters.
Lecturer: Dr. Josep Olm
Date: 13 April 2015
Number of Hours: 1
Credits: 0.2.
- Name: Regularization of two-fold bifurcations in planar piecewise-smooth systems.

Lecturer: Dr. John Hogan

Date: 23 June 2015

Number of Hours: 1.

Credits: 0.2.

- Name: Stochastic dynamics interrupted with large jumps at random times.
Lecturer: Shamik Gupta (Max Planck Institute for the Physics of Complex Systems, Dresden).
Date: 29 January 2016.
Number of Hours: 1
Credits: 0.2
- Name: Analysis and design of genetic control circuits for metabolism.
Lecturer: Diego Oyarzun (Mathematics, Imperial College London).
Date: 12 February 2016.
Number of Hours: 1
Credits: 0.2
- Name: Mathematical Modelling of the Steroidogenic Gene Regulatory Network in the Adrenal Gland
Lecturer: Dr. Eder Zavala (Mathematics, University of Exeter)
Date: 26 February 2016
Number of Hours: 1
Credits: 0.2
- Name: CMOS smart gas sensors, temperature sensors and IR devices
Lecturer: Prof. Florin Udrea (University of Cambridge, UK)
Date: 20 November 2016
Number of Hours: 3
Credits: 0.6

Training and Research Activities Report – Second Year

PhD in Information Technology and Electrical Engineering – XXIX Cycle

Christian Erazo Ordoñez

Student: Christian Erazo		Tutor: Mario di Bernardo		Cycle XXIX																						
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	Credits year 1						Credits year 2						Credits year 3													
	Estimated	1	2	3	4	5	6	Summary	Estimated	1	2	3	4	5	6	Summary	Estimated	1	2	3	4	5	6	Summary	Total	Check
Modules	20		7				2	9	15		10					10	21							0	19	30-70
Seminars	8					0,2		0,2	6	0,4	0,2			0,6	0,8	2	12							0	2,2	10-30
Research	32	9	7	6	7	7	6	42	39	7	5	7	8	4	8	39	30							0	81	80-140
	60	9	14	6	7	7,2	8	51	60	7,4	15	7	8	4,6	8,8	51	63	0	0	0	0	0	0	0	102	180

Year	Lecture/Activity	Type	Credits	Certification
1	Theory and applications of piecewise smooth systems.	Course	5	x
1	Convex Optimization.	Course	2	x
1	Three core issues for the Internet: things, security and economics.	Course	2	x
1	Seminars of research group SINCRO	Seminar	1	x
2	Dinamica e Controllo Nonlineare.	Course	6	x
2	Models, methods and software for Optimization.	Course	4	x
2	Mathematical Modelling of Atomic Force Microscopes.	Seminar	0,2	x
2	On Abel differential equations of the 2nd kind and exact inversion of boost DC/AC converters.	Seminar	0,2	x
2	Regularization of two-fold bifurcations in planar piecewise-smooth systems.	Seminar	0,2	x
2	Stochastic dynamics interrupted with large jumps at random times.	Seminar	0,2	x
2	Analysis and design of genetic control circuits for metabolism.	Seminar	0,2	x
2	Mathematical Modelling of the Steroidogenic Gene Regulatory Network in the Adrenal Gland.	Seminar	0,2	x
2	CMOS smart gas sensors, temperature sensors and IR devices.	Seminar	0,6	x

3. Research activity

Title: Dynamic cell-to-cell mapping algorithm for computing basins of attraction in bimodal Filippov Systems

The scope of my doctoral activity concerns with the study of piecewise non-smooth systems. In particular, I focused my research activities on the problem of computing basins of attraction in discontinuous systems, mostly known Filippov systems [1]. Many physical processes are interrupted by instantaneous events inducing discontinuities examples include, mechanical impact and friction oscillators [2], through power electronic and control systems with switches [3] or neuronal and biological models [4]. Different numerical approaches have been developed for investigating their complex dynamics, by playing direct numerical simulations or by computing bifurcation diagrams. Less attention has been given in the literature to the problem of computing numerically basins of attraction (BA) in Filippov systems.

During the first year of my PhD, we reviewed several methods for computing basins of attraction (BA) in Filippov systems [5-6]. Initially, we focused on estimating BA provided by polyhedral Lyapunov functions [7]. Based on these polyhedral functions and regarding that they typically provide very conservative estimates, we tried to derive a simple approach for studying whether the velocity vector pointed inwards on all boundaries of the closed set. In order to establish whether the region of interest is invariant with respect to the system trajectories. The results showed that just for systems having a fixed point of the star-type, the algorithm is able to construct invariant regions, while some inconvenient are present for systems which have different attractors.

According to this, in the second year, with the collaboration of Professors Martin Homer and Petri Piiroinen, I developed an algorithm based on the Simple Cell Mapping (SCM) method [8]. The use of discrete maps and Cell mapping methods is a very common methodology to compute basins of attraction in piecewise systems, examples are found in [9,10], where discrete maps are computed and iterated using Cell Mapping methods. Nevertheless, analytical solutions of discontinuous systems usually are difficult to obtain and accurate numerical integration routines are needed. Therefore, we developed an algorithm based on the Simple Cell Mapping (SCM) method which exploits the event-driven integration routine proposed in [11], that can cope with the presence of sliding solutions and automatically correct for possible numerical drifts. Our algorithm encompasses a dynamic selection of the cells. Specifically, after an initial application of SCM, layers of cells are added and examined iteratively. The mapping information is stored and used at each iteration, such that integrations for just the extra cells are performed. Moreover, a refinement stage is used to obtain a better resolution of the basin boundary. We illustrate the effectiveness of our algorithm by computing basins of attraction for Relay feedback systems, Sliding Control systems and non-smooth systems as reported in [13,14].

In the last months of the second year, with the collaboration of Professor Martin Homer and the PhD student Emanuel L., I focused on the study of Filippov solutions in co-dimension 2, where the sliding vector field is given by the intersection of the switching manifolds [15]. The aim is to extend the algorithm for computing basins of attraction in the case of multi-input multi-output systems. Specially, we are modeling the earthquake dynamics as a Filippov system, the aim is to develop a numerical tool for simulating and computing basins of attraction in high order Filippov systems which allows characterize their dynamics.

References:

- [1] A. F. Filippov and F. M. Arscott, *Differential equations with discontinuous righthand sides: control systems*, vol. 18, Springer Verlag, 1988.
- [2] P. Kowalczyk and P.T. Piiroinen. Two-parameter sliding bifurcations of periodic solutions in a dry-friction oscillator. *Physica D: Nonlinear Phenomena*, 237(8):1053 – 1073, 2008.
- [3] M Bernardo, C Budd, AR Champneys, P Kowalczyk, *Piecewise-smooth dynamical systems: theory and applications*.
- [4] A. Merola, C. Cosentino, and F. Amato, “An insight into tumor dormancy equilibrium via the analysis of its domain of attraction,” *Biomedical signal processing and control*, vol. 3, no. 3, pp. 212–219, 2008.
- [5] C. K. Luk, G. Chesi, and D. Han, “Guaranteed estimates of the domain of attraction for a class of hybrid systems,” in *IEEE 52nd Annual Conference on Decision and Control (CDC)*, Dec 2013, pp. 2024–2029.
- [6] M. Demenkov, “Estimating basin of attraction in piecewise-linear systems by nonsmooth lyapunov functions,” in *International Meeting on Analysis and Applications of Non-smooth Systems*, 2014.
- [7] Blanchini, Franco, and Stefano Miani. *Set-theoretic methods in control*. Boston: Birkhäuser, 2008.
- [8] Hsu, C. S., *Cell-to-cell mapping: a method of global analysis for nonlinear systems*, vol. 64, Springer Verlag 1987.
- [9] Joanna F. Mason, Petri T. Piiroinen, R. Eddie Wilson, and Martin E. Homer, Basins of attraction in nonsmooth models of gear rattle, *International Journal of Bifurcation and Chaos*, pp. 203–224. 2009.
- [10] Gyebrószki, G., and Csernák, G., *Methods for the Quick Analysis of Micro-chaos*. In *Applied Non-Linear Dynamical Systems* (pp. 383-395). Springer International Publishing. (2014).
- [11] Petri T Piiroinen and Yuri A Kuznetsov, An event-driven method to simulate filippov systems with accurate computing of sliding motions, *ACM Transactions on Mathematical Software (TOMS)*, 34,(2008), p. 13.
- [12] M. di Bernardo, C. Erazo, M. Homer and P. Piiroinen, Dynamic cell-to-cell mapping algorithm for computing basins of attraction in bimodal Filippov Systems, *Conference on open problems in nonsmooth dynamics*, Barcelona, Spain.

- [13] Dieci, L., & Difonzo, F., A comparison of Filippov sliding vector fields in codimension 2. *Journal of Computational and Applied Mathematics*, 262, 161-179, 2014.

Collaborations:

- Prof. Martin Homer, Faculty of Engineering University of Bristol.
- Dr. Petri Piiroinen, Faculty of University of Galway, Ireland.
- PhD student Emanuel Lorenzano, University of Bologna.

4. Products

a) Publications

I. Articles in preparation

- M. di Bernardo, C. Erazo, M. Homer and P. Piiroinen, Dynamic cell-to-cell mapping algorithm for computing basins of attraction in bimodal Filippov Systems (in preparation).

5. Conference and seminars

Poster presentation:

Title: Dynamic cell-to-cell mapping algorithm for computing basins of attraction in bimodal Filippov Systems.

Event: Conference on open problems in nonsmooth dynamics.

Location: Centre de Recerca Matemàtica (CRM), Barcelona, Spain.

Date: February 1 to 5, 2016.

http://www.crm.cat/en/Activities/Curs_2015-2016/Pages/CNonsmooth.aspx

6. Activity Abroad

Period of research at the Department of Engineering Mathematics of University of Bristol (United Kingdom) to collaborate with Prof. Martin Homer. From 13.01.2016 to 01.03.2016.

7. Tutorship

- Assistant for exercises of the B.Sc. course “Controlli automatici”, held by Prof. Mario di Bernardo, 4 hours.
- Assistant for exercises of the B.Sc. course “Dinamica e Controllo nonlineare”, held by Prof. Mario di Bernardo, 8 hours.