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Dynamic Cell-to Cell Mapping for computing basins of attraction in bimodal Filippov Systems

- Basins of attraction (BA) plays an important role in control systems since it provides a measure of how much the system can be perturbed until becomes unstable. There exists an extensive literature of BA in smooth systems, nevertheless, less results deal with the problem of computing BA in Filippov systems. Filippov systems are often used for modeling mechanical (earthquakes dynamics), electrical(power converters) and biological systems(predator – prey model).
- Within the community of control systems, optimal estimates of the BA are obtained by enlarging level sets of different kind of Lyapunov functions, however they are conservative.
- Here, we present an algorithm based on the Simple Cell Mapping (SCM) method which exploits the event-driven integration routine that can cope with the presence of sliding solutions and automatically correct for possible numerical drifts. Moreover, our algorithm encompasses a dynamic selection of the cell state space. In particular, after an initial application of SCM, layers of cells are added and examined iteratively. Finally a refinement stage is used to obtain a better resolution of the basin boundary.

Introduction

A bimodal Filippov system can be written as:

$$\dot{x} = \begin{cases} F_1(x), H(x) > 0 \\ F_2(x), H(x) < 0 \end{cases}$$

Where $F_1(x)$ and $F_2(x)$ are two smooth vector fields and $\mathrm{H}(x)$ is a smooth scalar function which zero set defines a smooth switching manifold Σ in $\mathbb{R}^2$ that is

$$\Sigma := \{ x \in \mathbb{R}^n : H(x) = 0 \}$$

with $\nabla H(x) \neq 0 \;\; \forall x \in \Sigma$.

Methodology

In simple cell mapping (SCM) [2], the state space $\mathbb{R}^N$ of a dynamical system is restricted to a bounded region Ω which is divided into cells. The state of the examined system is described with the index of the cell corresponding to that state. In SCM only one image cell is determined for each cell using the center point of the cell. The image cell corresponding to cell z is denoted by $C(z)$. The mapping $z(n+1) = C(z(n))$ with $C: \mathbb{N} \to \mathbb{N}$ is called SCM. In SCM two kinds of cells are distinguished.

- *Periodic cells*: for which $z = C^m(z)$ is true, for $m \in \mathbb{N}$. In this case z is a m - periodic cell.
- *Transient cells*: which are not periodic cells.

The main procedure of the SCM is determining the type and properties of every cell.

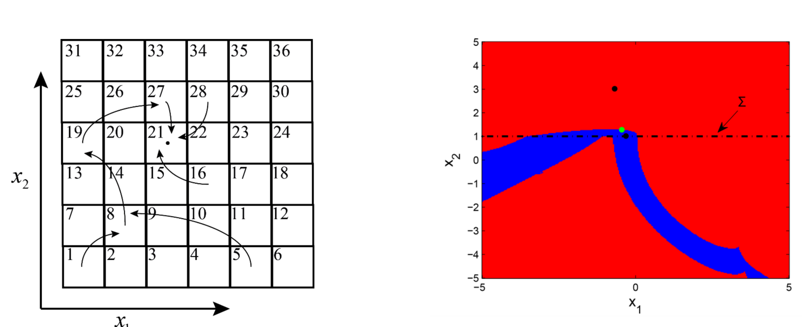
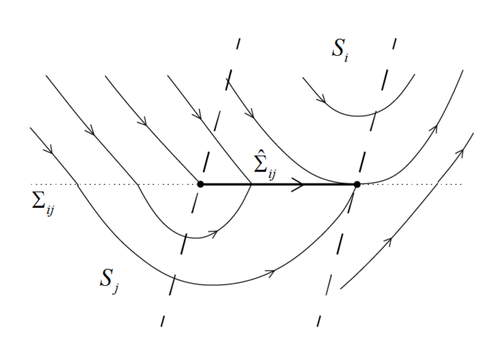


Figure: Cell state space, where it can identify a periodic cell with index 21. The cells 1, 5, 8, 19 , 27 and 28 are a periodic group.

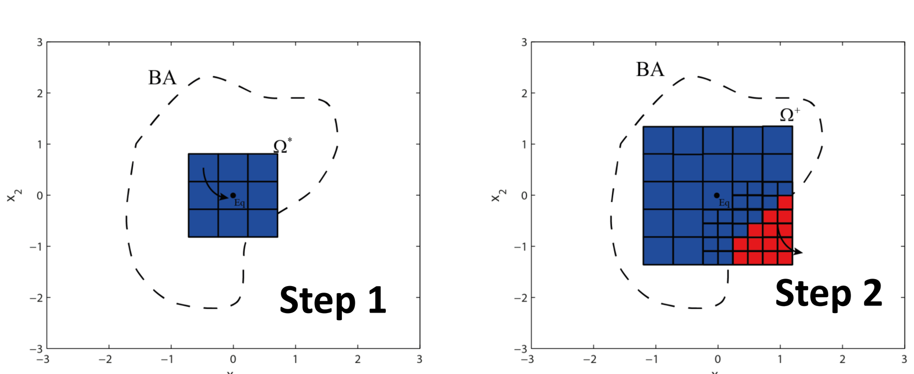
Results

- An event driven methodology is used to detect numerically Σ_{ij} and then switch the corresponding vector fields according to the system dynamics.
- In addition, due to the possibility of drifting away from the discontinuity surface, because of accumulation of numerical errors combined with the neutral stability of $\hat{\Sigma}$, a modified sliding vector field $\hat{F}_{ij}$ is introduced.



Cell-state space selection

- The idea is to use a set of cells $\boldsymbol{S_i} = \{\boldsymbol{z_1}, \boldsymbol{z_2}, .., \boldsymbol{z_n}\}$ in order to cover an invariant set of interest. Then by using the SCM, determine which cells lie to the basin of a given equilibrium **Eq**.
- In addition, if the given set of initial sets $\boldsymbol{S_i}$ is tagged as part of the basin of **Eq**, a new set of cells is added and examined in a second iteration of the algorithm, by using the stored information in the previous iteration
- Finally, a refinement stage is carried out by detecting the boundary cells, and reusing the SCM application.



Validation

A discontinuous control system is described by

$$\dot{x} = \begin{pmatrix} -1 & 1 \\ 0 & 3 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

where the control is express as

$$u = -10\,\mathrm{sgn}(x_1 + x_2)$$

The control strategy is designed to steer the system dynamics to the zero equilibrium. By rewriting the system in the Filippov form we get

$$\dot{x} = \begin{cases} F_1(x), H(x) > 0 \\ F_2(x), H(x) < 0 \end{cases}$$

where the two vector fields are:

$$F_1(x) = \begin{pmatrix} -x_1 + x_2 \\ 3x_2 - 10 \end{pmatrix}, \;\; F_2(x) = \begin{pmatrix} -x_1 + x_2 \\ 3x_2 + 10 \end{pmatrix}$$

The single switching manifold is defined as $\Sigma = \{x \in \mathbb{R}^2 : x_1 + x_2 = 0\}$

Sequence of the algorithm

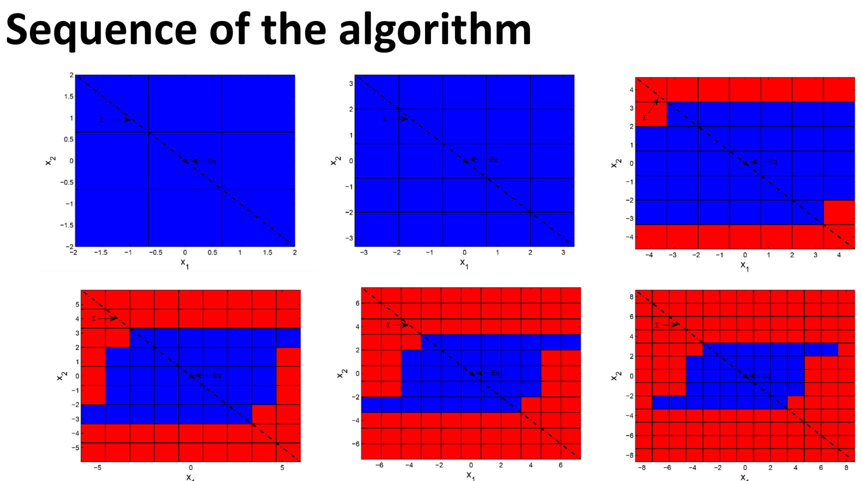


Figure: Numerical result of the sequence carried out by the algorithm, where blue regular cells correspond to the basin of the equilibrium **Eq.**

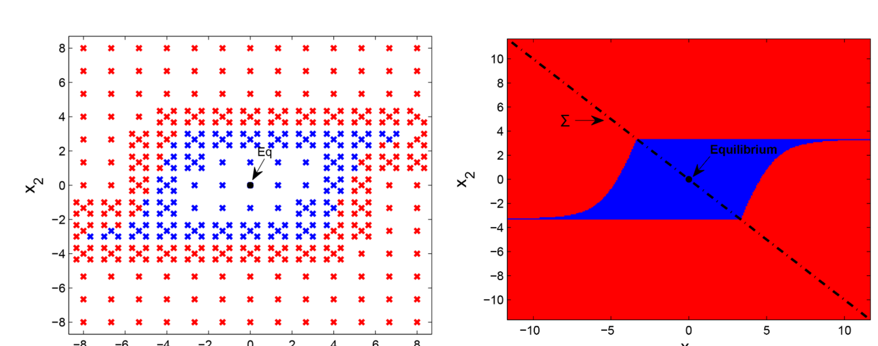


Figure: Left side plot shows the subdivision taken place on the basin boundary cells, while the final output window of the basins of attraction of **Eq** is on the right.

We develop this project with the collaboration of the professors:

- Martin Homer - Department of Engineering Mathematics, University of Bristol, United Kingdom.
- Petri Piiroinen - Department of Mathematical Physics, National University of Ireland, Galway.

We have developed a numerical tool for computing basins of attraction in bimodal Filippov systems. It has been evaluated for different discontinuous systems such as relay feedback systems, discontinuous control systems and also in piecewise continuous systems.
In the future, the scope of the work will be:

- To extend this method for computing BA for high order Filippov systems evolving more that one switching manifolds, in particular consider a choice of sliding vector field on the intersection of two co dimension 1 manifolds.
- To develop control strategies based on Cell-mapping methods, enlarging the basin of attraction.