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XXXII Cycle - I year presentation

Modelling and control of
coevolving dynamical networks
with multiple time-scales



General Information

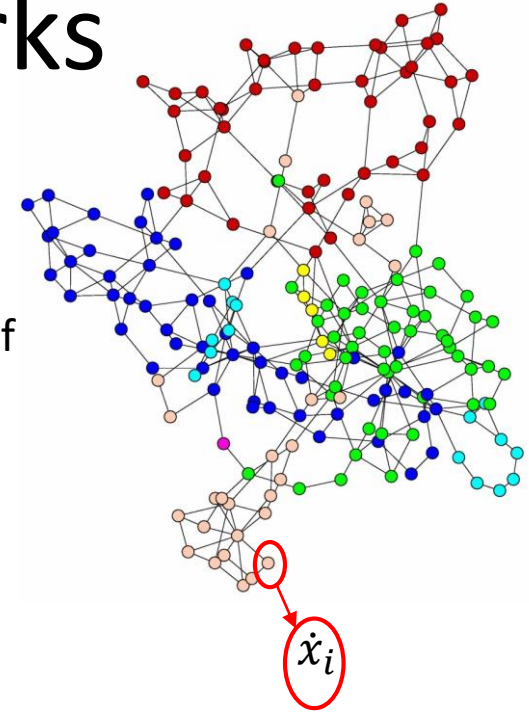
- Graduated in Management Engineering at Università di Napoli Federico II;
- Now, PhD student at SINCR0 Lab, Department of Information Technology and Electrical; Engineering
- M.I.U.R grant.



Complex networks

Def. COMPLEX NETWORK:

Networks whose structure is irregular, complex and dynamically evolving in time, with the main focus moving from the analysis of small networks to that of systems with thousands or millions of nodes, and with a renewed attention to the properties of networks of dynamical units.



$$\dot{x}_i = f(x_i) + c \sum_j \sigma_{ij} h(x_j - x_i)$$

individual dynamics

coupling term

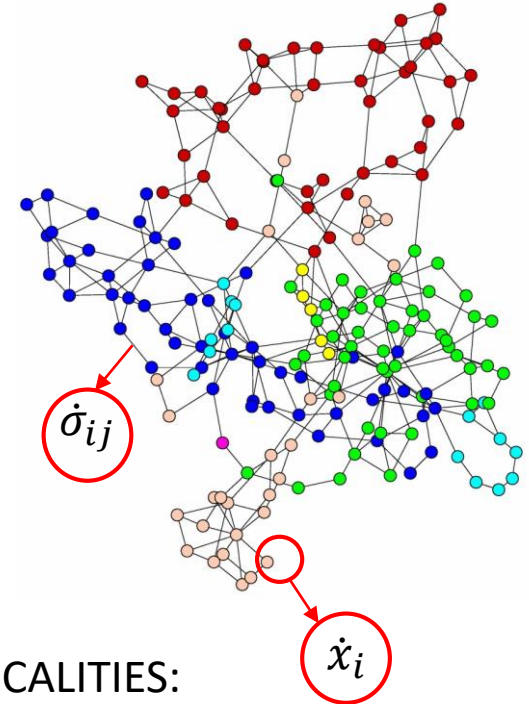
Co-evolving networks

Def. CO-EVOLVING NETWORKS:

Networks in which there is a strong interplay among the non-trivial structure and the edges' and the nodes' systems that **dynamically** evolve in time.

$$\dot{x}_i = f(x_i) + c \sum_j \sigma_{ij}(t) h(x_j - x_i)$$

$$\dot{\sigma}_{ij} = g(\sigma_{ij}, x_i, x_j)$$



CRITICALITIES:

- the whole network is a large system with a huge number of state variable;
- Few results on coevolving networks and no one on the networks in presence of multiple time-scales.

Co-evolving networks with multiple time-scales

Def. CO-EVOLVING NETWORKS WITH DIFFERENT TIME-SCALES:

Networks in which there is a strong interplay among the non-trivial structure and the edges' and the nodes' systems that **dynamically** evolve in time **with different time-scales**.

Steering complex network to a desired collective behavior!

ADVANTAGES:

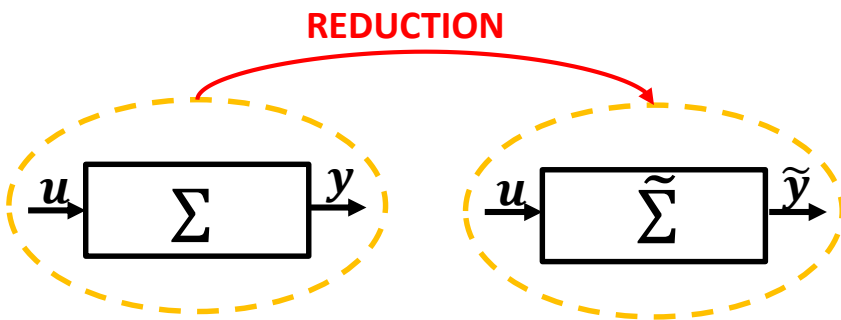
- Models capturing more aspects of the real phenomena;
- Reduction in complexity if the **hypothesis of different time scales** holds.



How to control a co-evolving network with multiple time-scales?

ORDER REDUCTION METHODS

Techniques to reduce the order of large scale system neglecting the presence of weakly observable/controllable system's modes.



SINGULAR PERTURBATION THEORY

Given a system:

$$\Sigma: \begin{cases} \dot{x} = f(x, \sigma) \\ \epsilon \dot{\sigma} = g(x, \sigma) \end{cases}$$

Analysis and control method to reduce the order of a large-scale system Σ in which holds the **hypothesis of different time scales**, slow and fast.

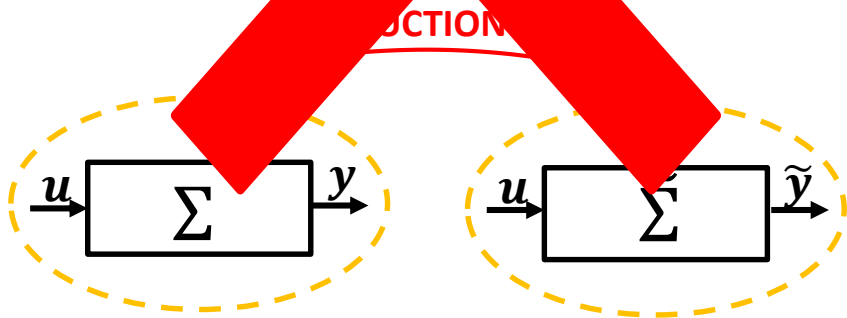
The perturbation parameter ϵ quantifies the time-scale separation.

This method allows us to **not completely ignore** the presence of a part of networks that could reduce the descriptive power of the system.

How to control a co-evolving network with multiple time-scales?

ORDER REDUCTION METHODS

Techniques to reduce the order of large scale systems neglecting the presence of weakly observable states available system's modes



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Possible investigations

- What are the features of the steady-state topologies?
- What are the collective behaviors arising in such networks?
- How can we control the whole network?
- What is the relation between the interaction topology and the dynamical systems?
- What is the optimal location of the input signal to achieve the desired collective behavior?
- ...



My papers

JOURNAL PAPERS:

1. DeLellis P., DiMeglio A., Garofalo F., Lo Iudice F. (2017). **“The evolving cobweb of relations among partially rational investors.”** *PloS one*, 12(2), e0171891.
2. DeLellis P., DiMeglio A., Garofalo F., Lo Iudice F. (2017). - De Lellis P., Di Meglio A., Lo Iudice F. (2017). **“Overconfident agents and evolving financial networks.”** *Nonlinear Dynamics*, 1-8.
3. DeLellis P., DiMeglio A., Garofalo F., Lo Iudice F. (2017). **“Steering opinion dynamics via containment control”.** *Computational social networks*, 4(1), 12.

CONFERENCE PAPERS:

1. DeLellis P., DiMeglio A., Garofalo F., Lo Iudice, F. (2017, May). **“Evolution of networks of financial agents driven by herding phenomena.”** In *American Control Conference (ACC), 2017* (pp. 1598-1603). IEEE.
2. De Lellis P., Di Meglio A., Lo Iudice F. (2016). **“Evolving topologies in artificial financial networks.”** Proceedings of the IEEE, COMPENG 2016.
3. DeLellis P., DiMeglio A., Garofalo F., Lo Iudice F. **“Containment control of networks with time-varying weights: an application to opinion dynamics in financial markets”.** 5th International Workshop COMPLEX NETWORKS AND THEIR APPLICATIONS.



Next year

| | Credits year 1 | | | | | | | | Credits year 2 | | | | | | | | Credits year 3 | | | | | | | | Total | Check | |
|----------|----------------|-----|-----|-----|-----|----|----|---------|----------------|---|---|---|---|---|---|---------|----------------|---|---|---|---|---|---|---------|-------|-------|--------|
| | Estimated | 1 | 2 | 3 | 4 | 5 | 6 | Summary | Estimated | 1 | 2 | 3 | 4 | 5 | 6 | Summary | Estimated | 1 | 2 | 3 | 4 | 5 | 6 | Summary | | | |
| Modules | 21 | 3 | 6 | 3 | 0 | 9 | 0 | 21 | 20 | | | | | | | | | | | | | | | | | | 30-70 |
| Seminars | 5 | 1,2 | 0,4 | 0,4 | 2,2 | 0 | 1 | 5,2 | 5 | | | | | | | | | | | | | | | | | | 10-30 |
| Research | 34 | 5,8 | 3,6 | 7 | 7,8 | 1 | 9 | 34 | 35 | | | | | | | | | | | | | | | | | | 80-140 |
| | 60 | 10 | 10 | 10 | 10 | 10 | 10 | 60 | 60 | | | | | | | | | | | | | | | | | | 180 |

NEXT OBJECTIVES:

1. Generalize the model of coevolving networks and find their properties;
2. Find the well-defined control framework for co-evolving complex networks;
3. Transmit our idea to various communities of interest.

How can we control a co-evolving network with multiple time-scales?

"t" TIME-SCALE

$$\begin{cases} \dot{x} = f(x, \sigma) \\ \epsilon \dot{\sigma} = g(x, \sigma) \end{cases}$$

FAST TIME-SCALE

$$\begin{cases} x' = \epsilon f(x, \sigma) \\ \sigma' = g(x, \sigma) \end{cases}$$

$$\begin{array}{c} \tau = t/\epsilon \\ \longleftrightarrow \\ 0 < \epsilon \ll 1 \end{array}$$

$\epsilon \rightarrow 0$

$\epsilon \rightarrow 0$

REDUCED MODEL (DAE)

$$\begin{cases} \dot{x} = f(x, \sigma) \\ 0 = g(x, \sigma) \end{cases}$$

BOUNDARY LAYER SYSTEM

$$\begin{cases} x' = 0 \\ \sigma' = g(x, \sigma) \end{cases}$$

CRITICAL MANIFOLD

M_0