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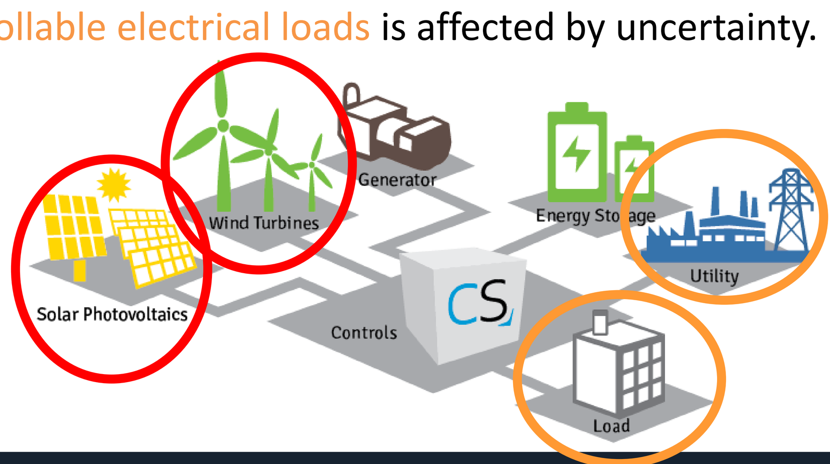
XXX Cycle - II year presentation

Probabilistic Short-term Forecasting Methods in Smart Grids

Why forecasting in Smart Grids

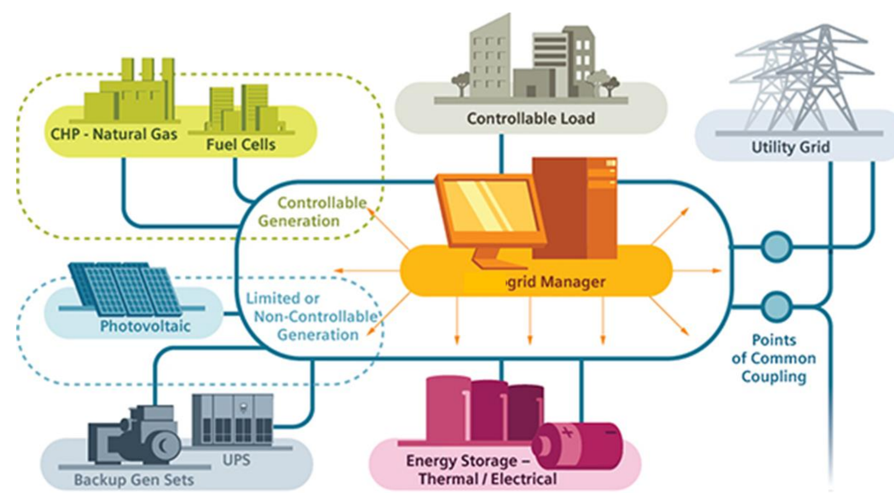
Electrical distribution systems are evolving towards the new concepts of Smart Grids. Their planning and management is a complex task, since non-programmable renewable power plants are characterized by a significant intrinsic randomness due the uncertainty affecting the corresponding main natural source. Also the power demand of non-controllable electrical loads is affected by uncertainty.

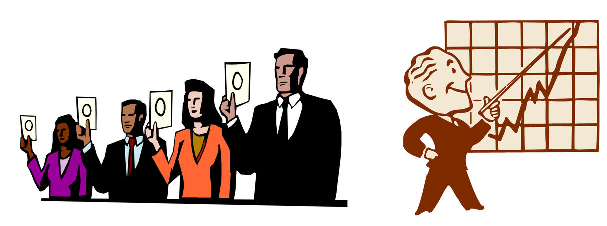


Why short-term time horizon

Few-minutes to some-hours forecasts are used in Smart Grids operation for optimally managing both power generation (unit commitment, dispatching) and load demand (load shedding and switching).

Also, short-term forecasts are mandatory in order to timely cope with power line congestions and to obtain short-term estimation of the reliability of power systems in extreme event conditions, such as extreme wind speeds.





Eventually, from an economic point of view, one-day to seven-day forecasts are used for optimizing electric market participation by energy producers and consumers.

A PROBABILISTIC ENSEMBLE METHOD FOR SHORT-TERM FORECASTING OF PHOTOVOLTAIC POWER

Theoretical discussion

This research activity dealt with a probabilistic method for the short-term forecasting of photovoltaic (PV) power, based on a competitive ensemble of different base predictors. Three probabilistic methods (Bayesian BM, quantile regression QM and Markov chain MM) were selected as base predictors in order to obtain an ensemble of the predictive distribution with optimal characteristics of sharpness and reliability.

BAYESIAN BASE PREDICTOR

A Beta distribution with mean μ_{BM_h} and shape parameter σ_h models the PV power at the desired time horizon h

An underlying ARIMAX deterministic model provides an estimation of μ_{BM_h}

$$\mu_{BM_h} = \theta_0 + \varphi_1 P_{h-k} + \dots + \varphi_{p+d} P_{h-k-p-d+1} - +\theta_1 e_{h-k} - \dots - \theta_q e_{h-k-q+1} + e_h + +\lambda_1^{(1)} y_{h-k}^{(1)} + \dots + \lambda_p^{(1)} y_{h-k-p+1}^{(1)} + \dots \dots + \lambda_1^{(r)} y_{h-k}^{(r)} + \dots + \lambda_p^{(r)} y_{h-k-p+1}^{(r)}$$

Bayes' formula provides estimations of the shape parameter σ_h, given the dataset $\boldsymbol{P}_{h,k}$ collected until the forecast origin time $h-k$

$$p(\sigma_h|\boldsymbol{P}_{h,k},\bar{\boldsymbol{z}}_\sigma) = \frac{p(\boldsymbol{P}_{h,k}|\sigma_h)\cdot p(\sigma_h|\bar{z}_{1_\sigma},\dots,\bar{z}_{HP_\sigma})}{\int p(\boldsymbol{P}_{h,k}|\sigma_h)\cdot p(\sigma_h|\bar{z}_{1_\sigma},\dots,\bar{z}_{HP_\sigma})\cdot d\sigma_h}$$

QUANTILE REGRESSION BASE PREDICTOR

The generic α-quantile of PV power is modeled through a linear regression

$$P_h^{(\alpha)} = \boldsymbol{\beta}^{(\alpha)}\cdot \boldsymbol{y}_h + r_h$$

The expected value is $\hat{P}_h^{(\alpha)} = \hat{\boldsymbol{\beta}}^{(\alpha)}\cdot \boldsymbol{y}_h$

Given D past measurements, estimations $\hat{\boldsymbol{\beta}}^{(\alpha)}$ are obtained as

$$\hat{\boldsymbol{\beta}}^{(\alpha)} = \arg\min_{\boldsymbol{\beta}^{(\alpha)}} \sum_{d=1}^{D} I_d \text{, with } I_d = \begin{cases}(P_d - \boldsymbol{\beta}^{(\alpha)}\cdot\boldsymbol{y}_d)\cdot(\alpha-1) & if\ P_d < \boldsymbol{\beta}^{(\alpha)}\cdot\boldsymbol{y}_d \\ (P_d - \boldsymbol{\beta}^{(\alpha)}\cdot\boldsymbol{y}_d)\cdot\alpha & if\ P_d \geq \boldsymbol{\beta}^{(\alpha)}\cdot\boldsymbol{y}_d\end{cases}$$

MARKOV CHAIN BASE PREDICTOR

PV powers are classified in states that are assumed independent after the second time step:

$$p(S_h|S_{h-k},S_{h-2k},\dots,S_1) = p(S_h|S_{h-k},S_{h-2k})$$

Predicted probability of the i^{th} state

$$\pi_{h_i} = \sum_{j=1}^{N_S}\sum_{l=1}^{N_S} \hat{a}_{ijl}\cdot\pi_{h-k_j}\cdot\pi_{h-2k_l}$$

a_{ijl} is the probability $p(S_{h_i}|S_{h-k_j},S_{h-2k_l})$; its maximum likelihood estimation is $\hat{a}_{ijl} = \frac{n_{ijl}}{\sum_{i=1}^{N_S} n_{ijl}}$, where n_{ijl} is the number of transitions between states $i \to j \to l$

ENSEMBLE MODEL

Cumulative density functions of base predictors are combined in a linear pool

$$F_{LPE_{h,k}}(P_h) = \sum_{n=1}^{N} w_n \cdot F_{n_{h,k}}(P_h)$$

Linear pooling may provide over-dispersed forecasts, e.g., when weights are found by minimizing the continuous ranked probability score (CRPS, a proper score) in a single-objective (SO) procedure

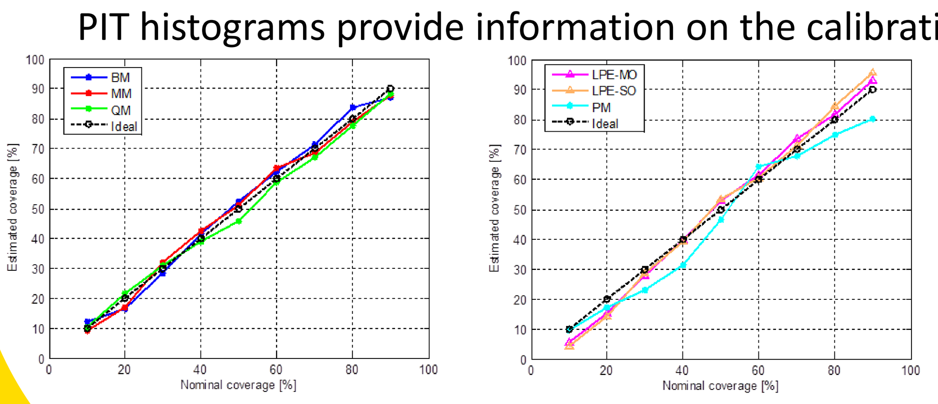
$$CRPS_h(F_h) = \int_0^{P_r} [F_h(P_h) - \mathrm{H}(P_h - P_h^*)]^2 \cdot dP_h$$

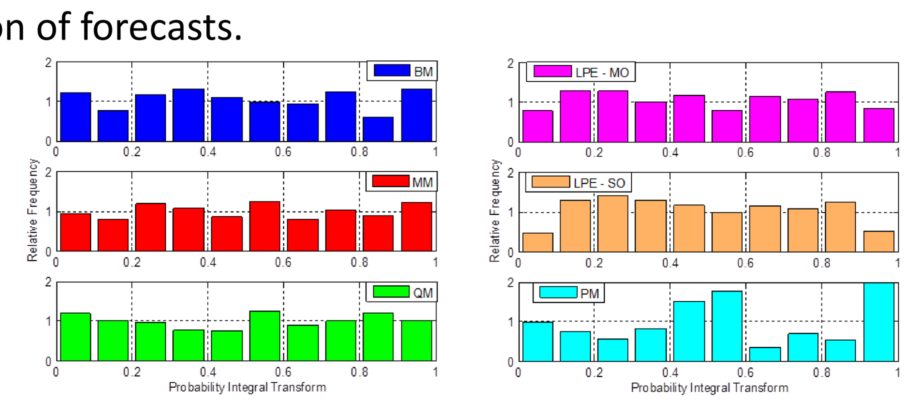
The over-dispersion of ensemble forecasts was overcome by minimizing the CRPS ϕ_1 and also the reliability index ϕ_2 in a multi-objective (MO) optimization procedure

$$\min_{\boldsymbol{w}} [\phi_1(F_{LPE_{h,k}}|w_1,\dots w_N), \phi_2(F_{LPE_{h,k}}|w_1,\dots w_N)]$$
$$s.t. \quad w_n \geq 0 \quad \forall n, \sum_{n=1}^{N} w_n = 1$$

Numerical results

The results of one-month forecasts made for a lead time k=1 hour are shown. Reliability diagrams and PIT histograms provide information on the calibration of forecasts.





Ensemble method	BM weight [-]	MM weight [-]	QM weight [-]
LPE - MO	0.060	0.265	0.675

Index	BM	MM	QM	PM	LPE - MO	LPE - SO
CRPS ϕ_1 [kW]	6.50	6.82	5.72	10.63	5.53	5.24
Reliability index ϕ_2 [%]	4.08	3.83	4.34	10.46	4.84	6.12

INVERSE BURR MODEL FOR EXTREME WIND SPEEDS

Theoretical discussion

This research activity dealt with an Inverse Burr (IB) distribution for the probabilistic modeling of extreme values of wind speed, together with several parameter estimation procedures. The reliability of an IB stress-strength (SS) model was then estimated in maximum likelihood (ML) and Bayesian frameworks.

Inverse Burr probability density function

$$f(x|\tau,\beta,\gamma) = \frac{\gamma\beta\tau^\beta}{\left[1+\left(\frac{\tau}{x}\right)^\beta\right]^{\gamma+1} x^{\beta+1}}$$

Inverse Burr cumulative density function

$$F(x|\tau,\beta,\gamma) = \frac{1}{\left[1+\left(\frac{\tau}{x}\right)^\beta\right]^\gamma}$$

Maximum likelihood estimation

$$\begin{cases}\frac{N\beta}{\tau} - (\gamma+1)\sum_{i=1}^{N}\frac{\beta\tau^{\beta-1}}{\left[1+\left(\frac{\tau}{x_i}\right)^\beta\right]x_i^\beta} = 0 \\ \frac{N}{\beta} + N\log\tau - (\gamma+1)\sum_{i=1}^{N}\frac{\tau^\beta(\log\tau - \log x_i)}{\left[1+\left(\frac{\tau}{x_i}\right)^\beta\right]x_i^\beta} - \sum_{i=1}^{N}\log x_i = 0 \\ \frac{N}{\gamma} - \sum_{i=1}^{N}\log\left[1+\left(\frac{\tau}{x_i}\right)^\beta\right] = 0\end{cases}$$

Moment estimation

$$\begin{cases}\frac{1}{N}\sum_{i=1}^{N} x_i - \gamma\tau\mathrm{B}\left(\gamma+\frac{1}{\beta},1-\frac{1}{\beta}\right) = 0 \\ \frac{1}{N}\sum_{i=1}^{N} x_i^2 - \gamma\tau^2\mathrm{B}\left(\gamma+\frac{2}{\beta},1-\frac{2}{\beta}\right) = 0 \\ \frac{1}{N}\sum_{i=1}^{N} x_i^3 - \gamma\tau^3\mathrm{B}\left(\gamma+\frac{3}{\beta},1-\frac{3}{\beta}\right) = 0\end{cases}$$

Quantile estimation

$$\begin{cases}x_{q_1}^* - \tau\left[\left(\frac{1}{q_1^*}\right)^{\frac{1}{\gamma}} - 1\right]^{-\frac{1}{\beta}} = 0 \\ x_{q_2}^* - \tau\left[\left(\frac{1}{q_2^*}\right)^{\frac{1}{\gamma}} - 1\right]^{-\frac{1}{\beta}} = 0 \\ x_{q_3}^* - \tau\left[\left(\frac{1}{q_2^*}\right)^{\frac{1}{\gamma}} - 1\right]^{-\frac{1}{\beta}} = 0\end{cases}$$

Reliability of the IB SS model

$$R_t = \int_0^{+\infty} g(y)\,P(Y<X|X=x)dy = = \int_0^{+\infty} g(y)\,[1-F(y)]dy = \frac{\gamma}{\gamma+\chi}$$

IB stress model $G(y|\beta,\chi) = \frac{1}{\left[1+\left(\frac{1}{y}\right)^\beta\right]^\chi}$

IB strength model $F(x|\beta,\gamma) = \frac{1}{\left[1+\left(\frac{1}{x}\right)^\beta\right]^\gamma}$

Numerical results

The Inverse Burr is compared to existing extreme-value distributions (Gumbel and Inverse Weibull) in terms of Kolmogorov-Smirnov and Chi-square tests for seven datasets. The Inverse Burr was the best fit in five cases.

Dataset and estimation procedure	Test statistics for estimated distributions								
	Gumbel			Inverse Weibull			Inverse Burr		
	KS-stat	χ^2-stat	χ^2-dof	KS-stat	χ^2-stat	χ^2-dof	KS-stat	χ^2-stat	χ^2-dof
D1 - MLE	0.153	2.745	2	0.174	3.543	2	0.152	3.016	2
D1 - ME	0.144	3.034	2	0.185	7.810	3	0.118	6.713	3
D1 - QE	0.148	1.852	1	0.170	2.731	1	0.125	0.121	1

The absolute bias (AB), mean square error (MSE) and mean absolute relative error (MARE) for synthetic samples proved the efficiency (EFF) of the Bayesian reliability estimation, specially for small datasets.

Index	Sample size				
	$N=1$	$N=3$	$N=10$	$N=15$	$N=30$
AB_{MLE}	0.0819	0.0260	0.0078	0.0040	0.0025
AB_{BE}	-0.0001	0.0001	0.0001	-0.0003	-0.0000
MSE_{MLE}	0.0495	0.0097	0.0018	0.0011	0.0005
MSE_{BE}	0.0003	0.0003	0.0003	0.0002	0.0002
EFF_{MSE}	177.2784	35.8229	6.9842	4.4339	2.3344
$MARE_{MLE}$	1.5160	0.7094	0.3424	0.2714	0.1899
$MARE_{BE}$	0.1467	0.1450	0.1401	0.1370	0.1286
EFF_{MARE}	10.3314	4.8930	2.4441	1.9818	1.4773

COLLABORATIONS AND PROJECTS

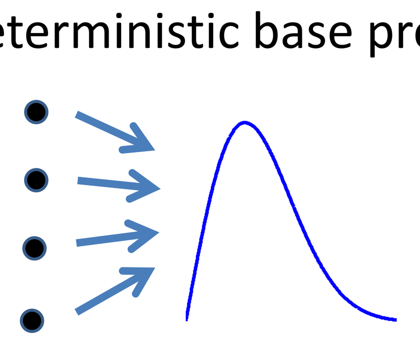
Università degli Studi di Napoli "Parthenope"

Politecnico di Torino

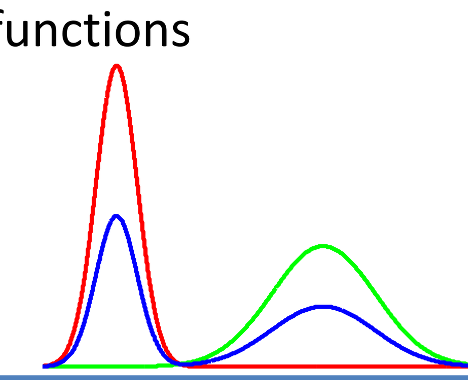
PROGETTO DI RICERCA PON03PE_00178_1 Microgrid in corrente continua ed alternata "M.I.C.C.A."

Next year developments:

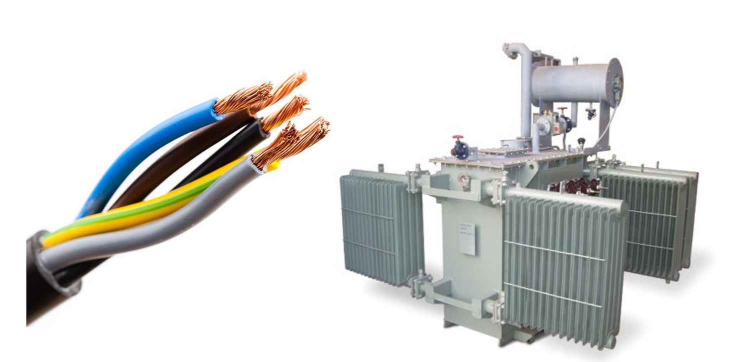
Probabilistic ensemble of deterministic base predictors



Estimation algorithms for parameters and confidence intervals of mixture probability density functions



Forecasting of dynamic thermal rating of Smart Grids components



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